

UNIT 4

Proportional Reasoning

Chapter 7

Ratios and Proportions

Chapter 8

Applying Percent

Chapter 9

Probability

In Unit 3, you learned about percents. In this unit, you will solve real-life problems involving proportions and percents. You will also discover how percents are used to describe probabilities.



INTERDISCIPLINARY PROJECT

Step Right Up and Win A Prize

Math and Recreation What are my chances of winning? You probably ask yourself that question any time you play a game that offers a prize for winning. You're about to embark on a gaming adventure. You'll investigate the mathematical probabilities of winning various carnival games. You'll also research and design a game of your own. So, come on and take a chance! Sharpen up that hand-eye coordination and grab your probability tool kit. This adventure is a win-win situation!



Log on to msmath2.net/webquest to begin your WebQuest.

Ratios and Proportions



How was math used to build Mount Rushmore?

To build Mount Rushmore, carvers used large models of the former presidents' heads that had a ratio of 1:12. So, one inch on the models equaled 12 inches, or one foot, on the mountain. They used proportions to determine the actual measures of the sculptured figures. In mathematics, you will use ratios and proportions to solve many real-life problems.

You will solve problems about scale models in Lesson 7-4.

GETTING STARTED

► Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 7. Refer to the lesson number in parentheses for review.

Vocabulary Review

Complete each sentence.

1. A comparison of two numbers by division is a _____. (Lesson 5-5)
2. A ratio that compares a number to 100 is called a _____. (Lesson 5-5)

Prerequisite Skills

Evaluate each expression. Round to the nearest tenth if necessary. (Lesson 1-3)

3. $100 \times 25 \div 52$
4. $10 \div 4 \times 31$
5. $\frac{63 \times 4}{34}$
6. $\frac{2 \times 100}{68}$

Write each fraction in simplest form.

(Lesson 5-3)

7. $\frac{9}{45}$
8. $\frac{16}{24}$
9. $\frac{38}{46}$

Write each decimal as a fraction in simplest form. (Lesson 5-4)

10. 0.78
11. 0.320
12. 0.06

Solve each equation. (Lesson 6-5)

13. $\frac{1}{4}x = 6$
14. $\frac{a}{5} = 8$
15. $4r = 22$
16. $2 = \frac{3}{5}z$

Complete. (Lesson 6-7)

17. 2 yd = ? ft
18. 48 oz = ? lb
19. $4\frac{1}{2}$ ft = ? yd
20. $3\frac{1}{4}$ h = ? min

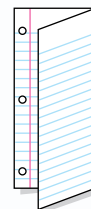
FOLDABLES Study Organizer

Ratios and Proportions

Make this Foldable to help you organize your notes. Begin with a sheet of notebook paper.

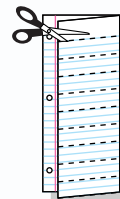
STEP 1

Fold
Fold lengthwise to the holes.



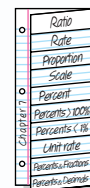
STEP 2

Cut
Cut along the top line and then make equal cuts to form 10 tabs.



STEP 3

Label
Label the major topics as shown.



Noteables™

Chapter Notes Each time you find this logo throughout the chapter, use your *Noteables™*: *Interactive Study Notebook with Foldables™* or your own notebook to take notes. Begin your chapter notes with this Foldable activity.



Readiness To prepare yourself for this chapter with another quiz, visit msmath2.net/chapter_readiness

What You'll LEARN

Write ratios as fractions and determine whether two ratios are equivalent.

NEW Vocabulary

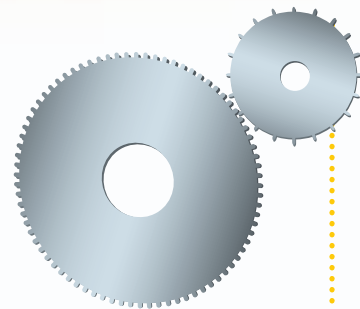
equivalent ratios

REVIEW Vocabulary

ratio: a comparison of two numbers by division
(Lesson 5-5)

WHEN am I ever going to use this?

GEARS Gears are used in objects that have spinning parts, such as bicycles and pendulum clocks. Suppose you have a larger gear that has 80 teeth being turned by a smaller gear that has 20 teeth. The comparison of the size of the larger gear to the size of the smaller gear is called a *gear ratio*.



1. Express the gear ratio as a fraction. Then write it as a fraction with a denominator of 1.
2. How many times does the smaller gear turn for every turn of the larger gear?
3. Describe some of the possible sizes of two gears that have a gear ratio of 3 to 1.

In Lesson 5-5, you learned that ratios can be written in several different ways. Ratios can also be simplified as shown below.

$$80 \text{ teeth} : 20 \text{ teeth} = \frac{\overset{4}{\cancel{80} \text{ teeth}}}{\underset{1}{\cancel{20} \text{ teeth}}} = \frac{4}{1}$$

EXAMPLES Write Ratios in Simplest Form

Write each ratio as a fraction in simplest form.

1 25 to 10

$$\begin{aligned} 25 \text{ to } 10 &= \frac{25}{10} && \text{Write the ratio as a fraction.} \\ &= \frac{5}{2} && \text{Simplify.} \end{aligned}$$

Written as a fraction in simplest form, the ratio 25 to 10 is $\frac{5}{2}$.
For ratios, leave the fraction as an improper fraction.

1 6:18

$$\begin{aligned} 6:18 &= \frac{6}{18} && \text{Write the ratio as a fraction.} \\ &= \frac{1}{3} && \text{Simplify.} \end{aligned}$$

The ratio 6:18 is $\frac{1}{3}$ in simplest form.

Your Turn Write each ratio as a fraction in simplest form.

a. 9 to 12

b. 27:15

c. 8 to 56

READING
in the Content Area

For strategies in reading this lesson, visit
msmath2.net/reading.

When writing ratios comparing units of length, units of time, units of weight, and so on, both measures should have the same unit.

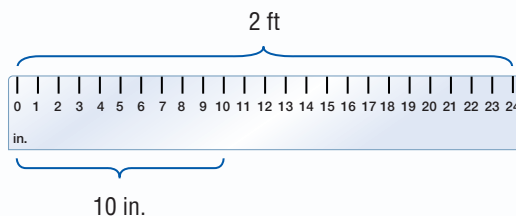
STUDY TIP

Canceling Units

You can cancel units that are the same in the numerator and in the denominator.

EXAMPLE Write a Ratio by Converting Units

- 1 Write the ratio 2 feet to 10 inches as a fraction in simplest form.



$$\frac{2 \text{ feet}}{10 \text{ inches}} = \frac{24 \text{ inches}}{10 \text{ inches}} \quad \text{Convert 2 feet to inches.}$$

$$= \frac{12}{5} \frac{24 \text{ inches}}{10 \text{ inches}} \quad \text{Divide by the GCF, 2 inches.}$$

$$= \frac{12}{5} \quad \text{Simplify.}$$

So, the ratio 2 feet to 10 inches can be written as $\frac{12}{5}$.

- 2 **Your Turn** Write each ratio as a fraction in simplest form.
- d. 4 feet : 4 yards e. 15 ounces to 3 pounds

Two ratios that have the same value are **equivalent ratios**.

EXAMPLES Compare Ratios

- 1 Determine whether 6 : 8 and 36 : 48 are equivalent.

Write each ratio as a fraction in simplest form.

$$6:8 = \frac{6 \div 2}{8 \div 2} = \frac{3}{4} \quad \text{The GCF of 6 and 8 is 2.} \quad 36:48 = \frac{36 \div 12}{48 \div 12} = \frac{3}{4} \quad \text{The GCF of 36 and 48 is 12.}$$

The ratios in simplest form both equal $\frac{3}{4}$. So, 6 : 8 and 36 : 48 are equivalent ratios.

- 2 **PONDS** It is recommended that for every 9 square feet of surface, a pond should have 2 fish. A pond that has a surface of 45 square feet contains 6 fish. Is this the correct number of fish based on the above recommendation?

$$9 \text{ to } 2 \text{ or } \frac{9}{2} \quad \text{Recommended ratio}$$

$$45 \text{ to } 6 \text{ or } \frac{45}{6} = \frac{15}{2} \quad \text{Actual ratio}$$

Since $\frac{9}{2} \neq \frac{15}{2}$, the ratios are not equivalent.

So, the number of fish is not correct for the pond.

- 3 **Your Turn** Determine whether the ratios are equivalent.

- f. $\frac{3}{8}$ and $\frac{6}{12}$ g. 35 students : 5 adults and 14 students : 2 adults

REAL-LIFE MATH

PONDS The recommended number of water lilies for a 9-square-foot pond is 1.



Skill and Concept Check

- Write the ratio 8 feet out of 15 feet in three different ways.
- OPEN ENDED** Write two different ratios that are equivalent to 3:2.
- Which One Doesn't Belong?** Identify the item that does not have the same ratio as the other three. Explain your reasoning.

$$\frac{30}{45}$$

4 to 6

26:39

3 to 2

- FIND THE ERROR** Marcus and Nicole are writing the ratio 2 days to 18 hours as a fraction in simplest form. Who is correct? Explain.

$$\begin{aligned} \text{Marcus} \\ 2:18 &= \frac{2}{18} \\ &= \frac{1}{9} \end{aligned}$$

$$\begin{aligned} \text{Nicole} \\ 48:18 &= \frac{48}{18} \\ &= \frac{8}{3} \end{aligned}$$

GUIDED PRACTICE

Write each ratio as a fraction in simplest form.

- 8:30
- 27 to 36
- 11 to 4
- 5 meters:1 meter
- 28 feet:7 yards
- 9 hours to 3 days

Determine whether the ratios are equivalent. Explain.

- $\frac{3}{5}$ and $\frac{9}{15}$
- 2:3 and 3:9
- HEIGHTS** Megan is 5 feet 3 inches tall, and her brother Troy is 5 feet 9 inches tall. Write a ratio in simplest form that compares Megan's height to Troy's height in inches.

Practice and Applications

Write each ratio as a fraction in simplest form.

- 24 to 9
- 14 to 70
- 18:19
- 45:21
- 120:30
- 33 to 90
- 66 inches:72 inches
- 12 pounds to 64 pounds
- 4 pounds to 12 ounces
- $8\frac{1}{3}$ yards:5 feet
- 18 weeks:1 year
- 45 minutes to 2 hours

Determine whether the ratios are equivalent. Explain.

- $\frac{6}{9}$ and $\frac{2}{3}$
- $\frac{9}{15}$ and $\frac{6}{10}$
- 4:7 and 16:49
- 8:21 and 16:42
- 14 in.:9 ft and 16 ft:12 yd
- 6 lb:72 oz and 2 lb:24 oz

HOMework HELP

For Exercises	See Examples
14–19	1, 2
20–25	3
26–31, 33–37	4, 5

Extra Practice
See pages 580, 602.

32. **WRITE A PROBLEM** Write a problem that could be represented by using a ratio.
33. **MOVIES** One week in 2002, a movie about the mathematician John Nash earned 6 million dollars at the box office. Total revenues for all movies that week were 78 million dollars. Write a ratio to compare the money earned by that movie to the money earned by all the movies.

PHOTOGRAPHS The *aspect ratio* of a photograph is a ratio comparing the length and width. A 35 mm negative has an aspect ratio of 1.5:1. Photo sizes with the same aspect ratio can be printed full frame without cropping. Determine which size photos can be printed full frame from a 35 mm negative.



34. $10'' \times 15''$ 35. $8'' \times 10''$ 36. $5'' \times 7.5''$

37. **MONEY** In simplest form, write the ratio of the cost in cents to time in years for each type of currency shown in the table. Are they equivalent? Explain.

Currency	Cost to Make	Time in Use
Sacagawea dollar coin	12¢	30 years
Dollar bill	3.5¢	18 months

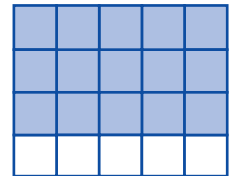
Source: www.sciencenews.org

38. **FIND A PATTERN** In decimal form, write the ratios of the first eight consecutive square numbers $1^2:2^2$, $2^2:3^2$, $3^2:4^2$, What do you notice about the ratios?
39. **CRITICAL THINKING** Find the next number in the pattern 480, 240, 80, 20, $\underline{\quad}$?. Explain your reasoning. (*Hint*: Look at the ratios of successive numbers.)

Spiral Review with Standardized Test Practice

40. **MULTIPLE CHOICE** Express the ratio of shaded squares to total squares as a fraction in simplest form.

Ⓐ $\frac{3}{4}$ Ⓑ $\frac{15}{20}$ Ⓒ $\frac{7}{10}$ Ⓓ $\frac{3}{5}$



41. **MULTIPLE CHOICE** In the seventh grade, 16 of the 120 students are left-handed. Write the ratio of left-handed students to right-handed students as a fraction in simplest form.

Ⓕ $\frac{2}{15}$ Ⓖ $\frac{2}{13}$ Ⓗ $\frac{13}{2}$ Ⓙ $\frac{15}{2}$

42. **TREES** A giant sequoia tree has a diameter of about 30 feet. What is the circumference? Round to the nearest foot. (Lesson 6-9)

Find the perimeter and area of each rectangle. (Lesson 6-8)

43. $\ell = 9$ ft, $w = 4$ ft 44. $\ell = 12$ cm, $w = 10.6$ cm
 45. $\ell = 7.5$ cm, $w = 2$ cm 46. $\ell = 1\frac{2}{5}$ in., $w = \frac{1}{2}$ in.

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Divide. Round to the nearest hundredth if necessary. (Page 562)

47. $9.8 \div 2$ 48. $\$4.30 \div 5$ 49. $\$12.49 \div 40$ 50. $27.36 \div 3.2$



HANDS-ON Mini Lab

Materials

- textbook
- computer
- stopwatch

What You'll LEARN

Determine unit rates.

NEW Vocabulary

rate
unit rate

Work with a partner.

Choose a page in a textbook and take turns typing as much as possible in 2 minutes.

1. Count the number of words that each of you typed.
2. Write the ratio *number of words to number of minutes* as a fraction.
3. Simplify the fractions by dividing the numerator and the denominator by 2.

A ratio that compares two quantities with different kinds of units is called a **rate**.

$$\frac{100 \text{ words}}{2 \text{ minutes}}$$

The units *words* and *minutes* are different.

When a rate is simplified so that it has a denominator of 1 unit, it is called a **unit rate**.

$$\frac{50 \text{ words}}{1 \text{ minute}}$$

The denominator is 1 unit.

The unit rate $\frac{50 \text{ words}}{1 \text{ minute}}$ can be read as *50 words per minute*. The table below shows some common unit rates used in everyday life.

Ratio	Unit Rate	Abbreviation	Name
$\frac{\text{number of miles}}{1 \text{ hour}}$	miles per hour	mi/h or mph	speed
$\frac{\text{number of miles}}{1 \text{ gallon}}$	miles per gallon	mi/gal or mpg	gas mileage
$\frac{\text{number of dollars}}{1 \text{ pound}}$	price per pound	dollars/lb	unit price
$\frac{\text{number of dollars}}{1 \text{ hour}}$	dollars per hour	dollars/h	hourly wage

EXAMPLE Find a Unit Rate

RUNNING Alethia ran 24 miles in 3 hours. What is her average speed in miles per hour?

Write the rate as a fraction. Then find an equivalent rate with a denominator of 1.

$$\begin{aligned} 24 \text{ miles in } 3 \text{ hours} &= \frac{24 \text{ mi}}{3 \text{ h}} \\ &= \frac{24 \text{ mi} \div 3}{3 \text{ h} \div 3} \\ &= \frac{8 \text{ mi}}{1 \text{ h}} \end{aligned}$$

Write the rate as a fraction.

Divide the numerator and the denominator by 3.

Simplify.

The average speed, or unit rate, is 8 miles per hour.



EXAMPLES Find Unit Rates

- 1 **GRID-IN TEST ITEM** Write 228 feet in 24 seconds as a unit rate in feet per second.

Read the Test Item

Write the ratio as a fraction. Then divide to get a denominator of 1.

Solve the Test Item

228 feet in 24 seconds

$$= \frac{228 \text{ ft}}{24 \text{ s}}$$

Write the rate as a fraction.

$$= \frac{228 \text{ ft} \div 24}{24 \text{ s} \div 24}$$

Divide the numerator and the denominator by 24.

$$= \frac{9.5 \text{ ft}}{1 \text{ s}}$$

Simplify.

The unit rate is 9.5 feet per second.

Fill in the Grid

			9	.	5
	/	/	/		
•	•	•	•	•	•
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	•
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	•	9	9	9

Test-Taking Tip

Grid-In Answers

To fill in the grid, first write the answers in the space above. Then shade the appropriate bubbles. You can align at the right as shown, or start in the left column.

- 1 Find the unit price per orange if it costs \$2 for six oranges. Round to the nearest hundredth if necessary.

$$\$2 \text{ for six oranges} = \frac{\$2}{6 \text{ oranges}}$$

Write the rate as a fraction.

$$= \frac{\$2 \div 6}{6 \text{ oranges} \div 6}$$

Divide the numerator and the denominator by 6.

$$\approx \frac{\$0.33}{1 \text{ orange}}$$

Simplify.

The unit price is about \$0.33 per orange.

- 2 **Your Turn** Find each unit rate. Round to the nearest hundredth if necessary.

a. 300 tickets in 6 days

b. 220 miles in 8 gallons

In Example 3, you found a special kind of unit rate, called the *unit price*. This is the price per unit and is useful when you want to compare the cost of an item that comes in different sizes.

EXAMPLE Choose the Best Buy

- 1 The costs of different sizes of peanut butter are shown at the right. Which jar costs the least per ounce?

Find the unit price, or the cost per ounce, of each jar. Divide the price by the number of ounces.

12-ounce jar $\$2.49 \div 12 \text{ ounces} \approx \0.21 per ounce

40-ounce jar $\$5.30 \div 40 \text{ ounces} \approx \0.13 per ounce

80-ounce jar $\$10.89 \div 80 \text{ ounces} \approx \0.14 per ounce

The 40-ounce jar costs the least per ounce.

Size	Price
12 oz	\$2.49
40 oz	\$5.30
80 oz	\$10.89

STUDY TIP

Estimation You can use compatible numbers to estimate the unit prices. For example, $\$2.49 \div 12 \text{ oz} \approx \$2.40 \div 12 \text{ oz}$, or $\$0.20/\text{oz}$.



Skill and Concept Check

- OPEN ENDED** Write a rate and then convert it to a unit rate.
- FIND THE ERROR** Mikasi and Julie are determining which size sports drink is the better buy per ounce: a 16-ounce bottle for \$1.95 or a 36-ounce bottle for \$3.05. Who is correct? Explain.

Mikasi
16-oz bottle: 12.2¢ per ounce
36-oz bottle: 8.5¢ per ounce

Julie
16-oz bottle: 8.2¢ per ounce
36-oz bottle: 11.8¢ per ounce

- NUMBER SENSE** In which situation will the rate $\frac{x \text{ ft}}{y \text{ min}}$ increase? Give an example to explain your reasoning.
 - x increases, y is unchanged
 - x is unchanged, y increases

GUIDED PRACTICE

Find each unit rate. Round to the nearest hundredth if necessary.

- 410 miles in 16 gallons
- 1,500 words in 25 minutes
- 5 pounds for \$2.49
- 3 cans of juice for \$2.95
- Which has the better unit price: a 6-pack of soda for \$2.99 or a 12-pack for \$4.50?
- MUSIC** Ethan can buy 4 CDs for \$71.96 at CD Express or 9 CDs for \$134.55 at Music Rox. Which has the better unit cost? Explain.

Practice and Applications

Find each unit rate. Round to the nearest hundredth if necessary.

- 360 miles in 6 hours
- 6,840 customers in 45 days
- 150 people for 5 classes
- \$7.40 for 5 pounds
- \$1.12 for 8.2 ounces
- 810 Calories in 3 servings
- 40 meters in 13 seconds
- 144 miles in 4.5 gallons

Choose the best unit price.

- \$3.99 for a 16-ounce bag of candy or \$2.99 for a 12-ounce bag
- aspirin sold in bottles of 50 for \$5.49, 100 for \$8.29, or 150 for \$11.99

- NUTRITION** Which soft drink described in the table has a lower amount of sodium per ounce? Explain.

Soft Drink	Serving Size (oz)	Sodium (mg)
A	12	70
B	8	35

- FOUNTAINS** The Prometheus Fountain in New York City pumps 60,000 gallons of water every 15 minutes. What is the unit rate?

HOMWORK HELP

For Exercises	See Examples
10–17, 21–23	1–3
18–20	4

Extra Practice
See pages 580, 602.

22. **WHALES** A humpback whale can migrate 3,000 miles in 30 days. Find a unit rate to describe the average speed per day during migration.
23. **POPULATION** The population of Arkansas is approximately 2.7 million people, and its land area is approximately 52,100 square miles. Find the *population density*, or the population per square mile.
24. **RESEARCH** Use the Internet or another source to find the population and land area of your state. Determine the population density.

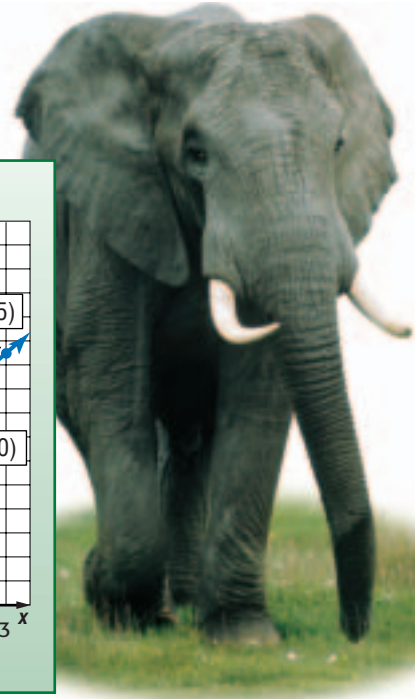
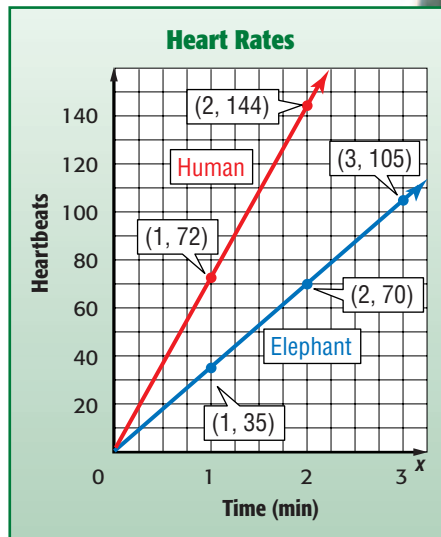


Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain by giving an example or a counterexample.

25. A ratio is a rate. 26. A rate is a ratio.

LIFE SCIENCE For Exercises 27 and 28, use the graph. It shows the average number of heartbeats for an adult elephant and an adult human.

27. Whose heart rate is greater? Explain.
28. Whose heart beats more times in one hour?
29. **CRITICAL THINKING** If 8 tickets are sold in 15 minutes, find the rate in tickets per hour.



Spiral Review with Standardized Test Practice

30. **MULTIPLE CHOICE** Which of the following has the lowest unit price?
 A 18 oz: \$5.40 B 16 oz: \$4.64 C 12 oz: \$3.72 D 10 oz: \$3.30
31. **MULTIPLE CHOICE** Which rate is the fastest?
 F 589 miles in 11 hours G 360 miles in 7 hours
 H 111 miles in 2 hours I 283 miles in 5 hours

Write each ratio as a fraction in simplest form. (Lesson 7-1)

32. 11:12 33. 24:4 34. 15 to 25 35. 20 to 14
36. A serving plate has a radius of 6 inches. Find the circumference to the nearest tenth. (Lesson 6-9)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Solve each equation. (Lesson 6-5)

37. $\frac{n}{8} = 7$ 38. $\frac{2}{3}x = 5$ 39. $9t = 12$ 40. $\frac{r}{5.2} = 4$



What You'll LEARN

Investigate rate of change.

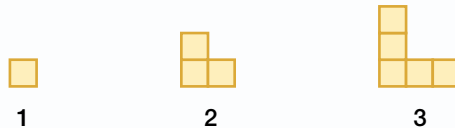
Materials

- square tiles
- grid paper

Rate of Change**INVESTIGATE** Work in groups of three.

A *rate of change* is a ratio that shows a change in one quantity with respect to a change in another quantity. In this lab, you will use tables and graphs to represent *constant* rates of change.

STEP 1 Use tiles to build the models shown below. Then continue the pattern to build the fourth and fifth models.



STEP 2 For each model, record the number of tiles and the perimeter in a table like the one shown at the right. The first two are done for you.

Model	Number of Tiles (x)	Perimeter (y)
1	1	4
2	3	8
3		
4		
5		

STEP 3 Draw a coordinate plane on grid paper and graph the ordered pairs (x, y) .

Writing Math

1. What do you notice about the points?
2. Find the ratio $\frac{\text{change in perimeter}}{\text{change in tiles}}$ between the second and third points, the third and fourth points, and the fourth and fifth points. Each ratio is a rate of change. **Describe** what you observe.
3. **Complete:** As the number of tiles increases by 2 units, the perimeter of the models increases by ? units.
4. Refer to the table at the right that appeared in Lesson 4-7. Find the ratio $\frac{\text{change in earnings}}{\text{change in hours worked}}$ for Greg and Monica. How do these values compare to the slopes that you found in Lesson 4-7?

Hours Worked	Earnings (\$)	
	Greg	Monica
1	4	5
2	8	10
3	12	15
4	16	20

STUDY TIP

Look Back You can review **slope** in Lesson 4-7.

5. **Make a conjecture** about the relationship between rate of change and a graph of the two quantities.

7-3

Algebra: Solving Proportions



What You'll LEARN

Solve proportions.

NEW Vocabulary

proportion
cross product

WHEN am I ever going to use this?

NUTRITION As part of a healthy diet, teens need 60 milligrams of vitamin C each day.

- Write the ratio $\frac{\text{vitamin C}}{\text{amount of cereal}}$ for a half-cup serving of cereal.
- Rewrite the ratio in Exercise 1 to find the unit rate in milligrams per cup.
- Simplify $\frac{60 \text{ mg}}{2 \text{ c}}$ to find the unit rate.
- Make a conjecture** about the rates $\frac{15 \text{ mg}}{0.5 \text{ c}}$ and $\frac{60 \text{ mg}}{2 \text{ c}}$.

Vitamin C (mg)	Amount of Cereal (c)
15	0.5
60	2

When two ratios are equivalent, they form a **proportion**. Since rates are types of ratios, they can also form proportions.

Noteables

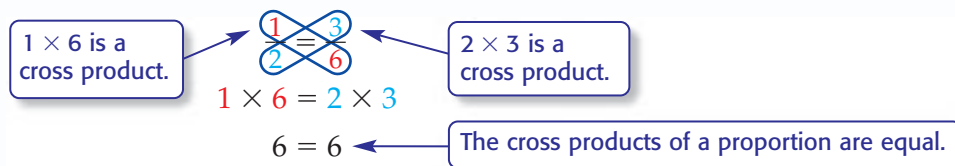
Key Concept: Proportions

Words A proportion is an equation stating that two ratios are equivalent.

Symbols

Arithmetic	Algebra
$\frac{1}{2} = \frac{3}{6}, \frac{8 \text{ ft}}{10 \text{ s}} = \frac{4 \text{ ft}}{5 \text{ s}}$	$\frac{a}{b} = \frac{c}{d}, \text{ where } b, d \neq 0$

In a proportion, a **cross product** is the product of the numerator of one ratio and the denominator of the other ratio.



EXAMPLE Identify a Proportion

1 Determine whether $\frac{6}{10}$ and $\frac{3}{5}$ form a proportion.

$\frac{6}{10} \stackrel{?}{=} \frac{3}{5}$ Write a proportion.

$6 \times 5 \stackrel{?}{=} 10 \times 3$ Find the cross products.

$30 = 30$ ✓ Multiply.

The cross products are equal, so the ratios form a proportion.



EXAMPLE**Identify a Proportion**

- 1 Determine whether $\frac{\$4}{9 \text{ oz}}$ and $\frac{\$21}{45 \text{ oz}}$ form a proportion.

$$\frac{4}{9} \stackrel{?}{=} \frac{21}{45} \quad \text{Write a proportion.}$$

$$4 \times 45 \stackrel{?}{=} 9 \times 21 \quad \text{Find the cross products.}$$

$$180 \neq 189 \quad \text{Multiply.}$$

The cross products are not equal, so the ratios do not form a proportion.

- 2 **Your Turn** Determine whether each pair of ratios forms a proportion.

a. $\frac{4}{9}$ and $\frac{2}{3}$ b. $\frac{15}{9}$ and $\frac{10}{6}$ c. $\frac{8 \text{ in.}}{21 \text{ s}}$ and $\frac{12 \text{ in.}}{31.5 \text{ s}}$

You can use cross products to find a missing term in a proportion. This is known as *solving the proportion*. Solving a proportion is similar to solving an equation.

EXAMPLES**Solve Proportions**

- 1 Solve $\frac{2.6}{13} = \frac{8}{n}$.

$$\frac{2.6}{13} = \frac{8}{n} \quad \text{Write the proportion.}$$

$$2.6 \cdot n = 13 \cdot 8 \quad \text{Find the cross products.}$$

$$2.6n = 104 \quad \text{Multiply.}$$

$$\frac{2.6n}{2.6} = \frac{104}{2.6} \quad \text{Divide each side by 2.6.}$$

$$n = 40 \quad \text{Simplify.}$$

The solution is 40.

- 2 **LIFE SCIENCE** The skull length of a crocodile is proportional to its body length. In one species, a 6-foot crocodile has a 2-foot skull. Estimate the length of a crocodile of that species that has a 3.5-foot skull.

$$\begin{array}{l} \text{body length} \rightarrow \frac{6 \text{ ft}}{2 \text{ ft}} = \frac{c \text{ ft}}{3.5 \text{ ft}} \\ \text{skull length} \rightarrow \end{array} \quad \text{Write a proportion.}$$

$$6 \cdot 3.5 = 2 \cdot c \quad \text{Find the cross products.}$$

$$21 = 2c \quad \text{Multiply.}$$

$$\frac{21}{2} = \frac{2c}{2} \quad \text{Divide each side by 2.}$$

$$10.5 = c \quad \text{Simplify.}$$

So, a crocodile with a 3.5-foot skull is about 10.5 feet long.

- 3 **Your Turn** Solve each proportion.

d. $\frac{1}{5} = \frac{x}{60}$ e. $\frac{10}{k} = \frac{2.5}{4}$ f. $\frac{2 \text{ ft}}{6 \text{ min}} = \frac{5 \text{ ft}}{t \text{ min}}$

REAL-LIFE CAREERS**How Does a Paleontologist Use Math?**

In 2001, paleontologists discovered a 6-foot skull of a prehistoric crocodile called *SuperCroc*. By using proportions, they estimated its total length as 40 feet.

Source: National Geographic

**Research**

For information about a career as a paleontologist, visit: msmath2.net/careers



Skill and Concept Check

- Writing Math** Explain how to determine if two ratios are equivalent.
- OPEN ENDED** Write a proportion that has $\frac{12}{15}$ as one of the ratios.
- Which One Doesn't Belong?** Identify the ratio that is not equivalent to the other three. Explain your reasoning.

$$\frac{2}{3}$$

$$\frac{6}{9}$$

$$\frac{4}{6}$$

$$\frac{2}{9}$$

GUIDED PRACTICE

Determine whether each pair of ratios forms a proportion.

4. $\frac{2}{3}$ and $\frac{6}{9}$

5. $\frac{16}{30}$ and $\frac{8}{10.5}$

6. $\frac{5 \text{ ft}}{7 \text{ s}}$ and $\frac{25 \text{ ft}}{49 \text{ s}}$

Solve each proportion.

7. $\frac{24}{6} = \frac{8}{x}$

8. $\frac{n}{5} = \frac{20}{25}$

9. $\frac{7}{t} = \frac{3}{9}$

10. $\frac{8}{12} = \frac{n}{3}$

11. $\frac{a}{6} = \frac{12}{8}$

12. $\frac{13}{25} = \frac{39}{r}$

13. $\frac{21}{5} = \frac{c}{7}$

14. $\frac{7}{x} = \frac{3.5}{9}$

15. **NUTRITION** An 8-ounce glass of orange juice contains 72 milligrams of vitamin C. How much juice contains 60 milligrams of vitamin C?

Practice and Applications

Determine whether each pair of ratios forms a proportion.

16. $\frac{1}{6}$ and $\frac{4}{24}$

17. $\frac{3}{9}$ and $\frac{2}{15}$

18. $\frac{27}{6}$ and $\frac{9}{2}$

19. $\frac{1}{4}$ and $\frac{4}{1}$

20. $\frac{\$8}{20 \text{ lb}}$ and $\frac{\$24}{60 \text{ lb}}$

21. $\frac{21 \text{ mi}}{6 \text{ h}}$ and $\frac{14.3 \text{ mi}}{3 \text{ h}}$

22. If there are 12 wheels, how many bicycles are there?
 23. If there are 32 paws, how many kittens are there?

Solve each proportion.

24. $\frac{5}{9} = \frac{10}{x}$

25. $\frac{2}{12} = \frac{a}{36}$

26. $\frac{16}{t} = \frac{2}{3}$

27. $\frac{n}{8} = \frac{3}{4}$

28. $\frac{c}{7} = \frac{18}{42}$

29. $\frac{30}{w} = \frac{8}{20}$

30. $\frac{9}{15} = \frac{b}{10}$

31. $\frac{3}{n} = \frac{27}{18}$

32. $\frac{x}{12} = \frac{12}{4}$

33. $\frac{45}{5} = \frac{t}{7}$

34. $\frac{6}{3} = \frac{5}{w}$

35. $\frac{y}{36} = \frac{15}{24}$

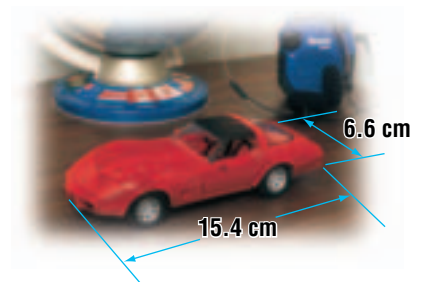
36. $\frac{350}{a} = \frac{2}{10}$

37. $\frac{380}{520} = \frac{760}{n}$

38. $\frac{3}{5} = \frac{0.2}{d}$

39. $\frac{2.5}{4.5} = \frac{7.5}{x}$

40. **MODELS** The toy car shown at the right is modeled after a real car. If the real car is 4.79 meters long, how wide is it? Write the width to the nearest hundredth of a meter.
41. **TESTS** Suppose you received a score of 13 out of 15 on a quiz. Write a proportion to find the percent. Then solve. Round to the nearest percent.



HOMESCHOOL HELP

For Exercises	See Examples
16–21	1, 2
22–46	3, 4

Extra Practice
See pages 580, 602.



COOKING For Exercises 42–44, refer to the table.

42. How many stalks of celery would you need to get $3\frac{1}{2}$ cups of celery slices?
43. How many cups of broccoli florets would 1 pound of broccoli produce?
44. **WRITE A PROBLEM** Write a real-life problem that can be solved using a proportion and one of the vegetable equivalents. Then write a proportion and solve.

Vegetable Equivalents	
$\frac{3}{4}$ lb broccoli	= 3 c florets
1 lb carrots	= 3 c slices
1 stalk celery	= $\frac{1}{2}$ c slices
2 peppers	= $2\frac{1}{2}$ c rings
1 lb potatoes	= $2\frac{3}{4}$ c cubes

45. **MULTI STEP** Amanda can word process 7 words in 6 seconds. At this rate, how many words can she word process in 3 minutes?
46. **MONEY** In 2002, France adopted the *Euro* as its currency, replacing the French franc. At that time, one Euro was worth \$0.90 in American currency. In Paris, France, a quart of milk cost 3 Euros. In Los Angeles, a quart of milk cost \$1.65. In which city was the quart of milk more expensive?



Data Update What is the current U.S. value of one Euro? Visit msmath2.net/data_update to learn more.

47. **CRITICAL THINKING** In the proportion $\frac{3}{6} = \frac{6}{12}$, the number 6 appears in two of the diagonal positions. This repeated number is called the *geometric mean* of the other two numbers. Find a pair of numbers other than 3 and 12 for which 6 is the geometric mean.

Spiral Review with Standardized Test Practice

48. **MULTIPLE CHOICE** Ming is paid an hourly rate. One week he earned \$157.50 by working 30 hours. If he works 35 hours the next week, how much will he earn?
- (A) \$135.00 (B) \$160.50 (C) \$183.75 (D) \$210.25
49. **GRID IN** The ratio of boys to girls in the seventh grade is 7 to 8. If there are 120 students in the seventh grade, how many of them are girls?
50. Choose the better unit price: 5 pounds of onions for \$2.99 or 3 pounds of onions for \$1.29. (Lesson 7-2)

Write each ratio as a fraction in simplest form. (Lesson 7-1)

51. 45:81 52. 49 to 14 53. 14 ounces to 5 pounds

MEASUREMENT Complete. (Lesson 6-7)

54. 5 qt = ? pt 55. $3\frac{1}{2}$ lb = ? oz 56. 28 c = ? qt

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Multiply. Write in simplest form. (Lesson 6-4)

57. $\frac{1}{5} \cdot 8$ 58. $1\frac{3}{4} \cdot 5$ 59. $16 \cdot 3\frac{1}{4}$ 60. $24 \cdot 1\frac{3}{8}$

What You'll LEARN

Use proportions to estimate.

Materials

- small bowl
- dried beans
- paper cup
- markers

Wildlife Sampling

INVESTIGATE Work in groups of three.

Naturalists can estimate the population in a wildlife preserve by using the *capture-recapture* technique. In this lab, you will model this technique using dried beans in a bowl to represent deer in a forest.



STEP 1 Fill a small bowl with dried beans.

STEP 2 Use the paper cup to scoop out some of the beans. These represent the original *captured* deer. Record the number in a table like the one shown at the right. Mark each bean with an X on both sides. Then return these beans to the bowl and mix well.

Original Number Captured _____			
Trial	Sample	Recaptured	<i>P</i>
A			
B			
C			
⋮			
J			
Total			

STEP 3 Scoop another cup of beans from the bowl and count them. This is the *sample* for Trial A. Count the beans with the X's. These are the *recaptured* deer. Record both numbers.

STEP 4 Use the proportion below to estimate the total number of beans in the bowl. This represents the total population *P*. Record the value of *P* in the table.

$$\frac{\text{captured}}{\text{total population } (P)} = \frac{\text{recaptured}}{\text{sample}}$$

STEP 5 Return all of the beans to the bowl.

STEP 6 Repeat Steps 3–5 nine times.

Writing Math

Work in groups of three.

1. Find the average of the estimates in column *P*. Is this a good estimate of the number of beans in the bowl? **Explain** your reasoning.
2. **Count** the actual number of beans in the bowl. How does this number compare to your estimate?

7-4a

Problem-Solving Strategy

A Preview of Lesson 7-4

Draw a Diagram

What You'll LEARN

Solve problems by drawing a diagram.

Mrs. Dixon said that we've gone about 90 miles, or $\frac{2}{3}$ of the way to the campsite.

So, how much farther do we have to go? I bet if we **draw a diagram**, we could figure it out.



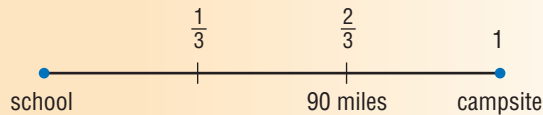
Explore

We know that 90 miles is about $\frac{2}{3}$ of the total distance.

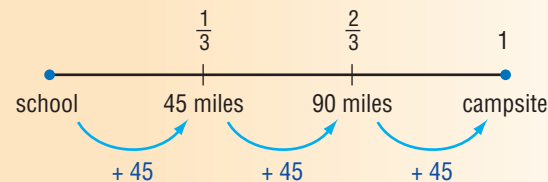
Plan

Let's draw a diagram showing the distance that we've already gone and the fractional part that it represents.

Solve



If $\frac{2}{3}$ of the distance is 90 miles, then $\frac{1}{3}$ of the distance would be 45 miles. So, the missing third must be another 45 miles.



The total distance is $90 + 45$ or 135 miles.

Examine

Since $\frac{2}{3}$ of the total distance equals 90 miles, the equation $\frac{2}{3}x = 90$ represents this problem. Solving, we get $x = 135$ miles. So, the solution checks.

Analyze the Strategy

1. **Explain** why drawing a diagram can be a useful problem-solving strategy.
2. **Determine** how far the trip would have been if 90 miles was only $\frac{1}{3}$ of the total distance. Draw a new diagram for this situation.
3. **Write** a problem that could be solved by drawing a diagram. Exchange your problem with a classmate and solve.

Apply the Strategy

Solve. Use the draw a diagram strategy.

4. **PHYSICAL SCIENCE** A ball is dropped from 10 feet above the ground. It hits the ground and bounces up half as high as it fell. This is true for each successive bounce. What height does the ball reach on the fourth bounce?
5. **FAMILY** At Nate's family reunion, 80% of the people are 18 years of age or older. Half of the remaining people are under 12 years old. If 20 children are under 12 years old, how many people are at the reunion?

Mixed Problem Solving

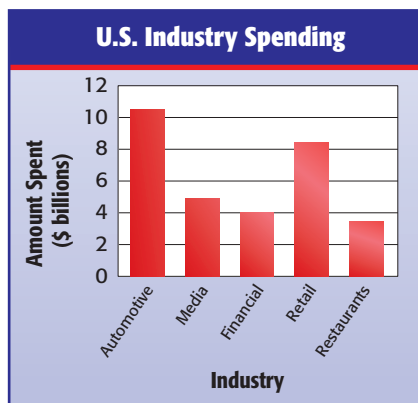
Solve. Use any strategy.

6. **TESTS** The scores on a social studies test are found by adding or subtracting points as shown at the right.

Answer	Points
Correct	+8
Incorrect	-4
No answer	-2

If Mario's score on a 15-question test was 86 points, how many of his answers were correct, incorrect, and blank?

7. **GAMES** Six members of a video game club are having a tournament. In the first round, every player will play a video game against every other player. How many games will be in the first round of the tournament?
8. **BUSINESS** The graph shows the annual spending by five industries.



Source: *The Top Ten of Everything*

Estimate how much more the automotive industry spends per year than the retail industry. Write in scientific notation.

9. **ENVIRONMENT** What fraction of landfills is paper and plastic packaging? Write in simplest form.

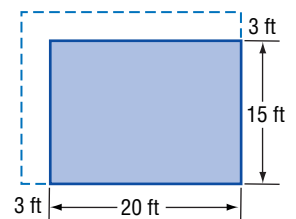


Source: ULS Report

10. **MONEY** Mr. Li has \$240 in his checking account after writing checks for \$15.70, \$43.20, and \$18. What was his balance before he wrote the three checks?

11. **STANDARDIZED TEST PRACTICE**

Danielle is adding 3 feet to the length and width of her rectangular garden, as shown at the right. Which expression represents the area of land that will be added to the garden?



- (A) $(15 + 3)(20 + 3)$
 (B) $(15 + 3)(20 + 3) - (15)(20)$
 (C) $(15 - 3)(20 - 3)$
 (D) $(15 - 3)(20 - 3) - (15)(20)$

HANDS-ON Mini Lab

What You'll LEARN

Solve problems involving scale drawings.

NEW Vocabulary

scale drawing
scale
scale factor
scale model

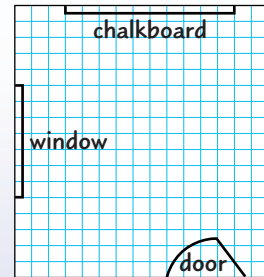
Work with a partner.

- Measure the length of each wall, door, window, and chalkboard in your classroom.
- Record each length to the nearest $\frac{1}{2}$ foot.

1. Let 1 unit on the grid paper represent 1 foot. So, 6 units = 6 feet. Convert all of your measurements to units.
2. On grid paper, make a drawing of your classroom like the one shown at the right.

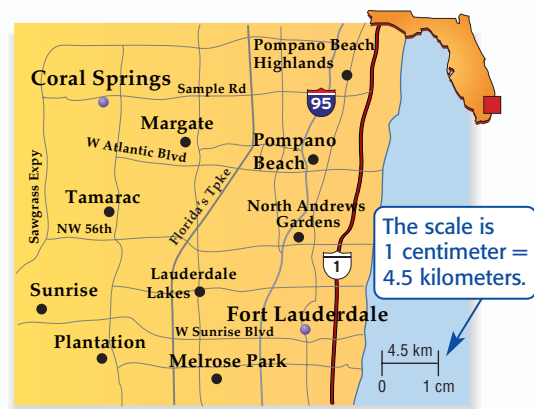
Materials

- measuring tape
- straightedge
- $\frac{1}{4}$ -inch grid paper



A **scale drawing** represents something that is too large or too small to be drawn at actual size.

The map is a scale drawing. Its **scale** gives the relationship between the distance on the map and the actual distance.



EXAMPLE Use a Scale Drawing

MAPS On the map above, the distance between Coral Springs and Fort Lauderdale is about 4.1 centimeters. What is the actual distance? Round to the nearest tenth.

Let d = the actual distance between the cities. Write and solve a proportion. Use the scale written as a fraction.

$$\begin{array}{rcc} & \text{Scale} & \text{Coral Springs to} \\ & & \text{Ft. Lauderdale} \\ \text{map} \rightarrow & \frac{1 \text{ centimeter}}{4.5 \text{ kilometers}} = & \frac{4.1 \text{ centimeters}}{d \text{ kilometers}} \quad \leftarrow \text{map} \\ \text{actual} \rightarrow & & \leftarrow \text{actual} \\ & 1 \cdot d = 4.5 \cdot 4.1 & \text{Cross products} \\ & d = 18.45 & \text{Simplify.} \end{array}$$

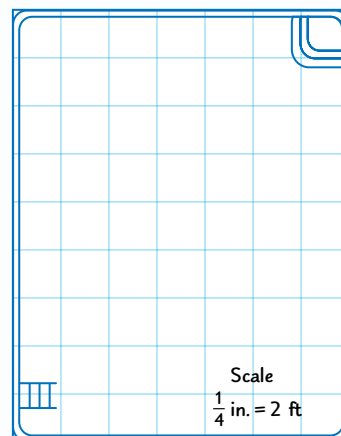
The distance between Coral Springs and Fort Lauderdale is about 18.5 kilometers.

A blueprint is another example of a scale drawing.

EXAMPLE Read a Scale Drawing

1 POOLS On the blueprint of the pool, each square has a side length of $\frac{1}{4}$ inch. What is the actual width of the pool?

The pool on the blueprint is $1\frac{3}{4}$ inches wide. Let w = the actual width of the pool. Write and solve a proportion using the scale.



$$\begin{array}{l} \text{Scale} \quad \text{Width of Pool} \\ \text{blueprint} \rightarrow \frac{\frac{1}{4} \text{ inch}}{2 \text{ feet}} = \frac{1\frac{3}{4} \text{ inches}}{w \text{ feet}} \quad \leftarrow \text{blueprint} \\ \text{actual} \rightarrow \end{array}$$

$$\frac{1}{4} \cdot w = 2 \cdot 1\frac{3}{4} \quad \text{Cross products}$$

$$\frac{1}{4}w = \frac{14}{4} \quad \text{Multiply.}$$

$$w = 14 \quad \text{Simplify. Multiply each side by 4.}$$

The width of the pool is 14 feet.

In Lesson 7-1, you simplified ratios by converting units. You can use the same method to simplify the scale in Example 2. A scale written as a ratio in simplest form is called the **scale factor**.

$$\begin{aligned} \text{scale} \rightarrow \frac{\frac{1}{4} \text{ inch}}{2 \text{ feet}} &= \frac{\frac{1}{4} \text{ inch}}{24 \text{ inches}} && \text{Convert 2 feet to inches.} \\ &= \frac{\frac{4}{4} \cdot \frac{1}{4} \text{ inch}}{24 \text{ inches}} && \text{Multiply by } \frac{4}{4} \text{ to eliminate the} \\ &= \frac{1}{96} && \text{fraction in the numerator.} \end{aligned}$$

\leftarrow scale factor

EXAMPLE Find the Scale Factor

1 Find the scale factor of a blueprint if the scale is 1 inch = 8 feet.

Write the ratio of 1 inch to 8 feet in simplest form.

$$\frac{1 \text{ inch}}{8 \text{ feet}} = \frac{1 \text{ inch}}{96 \text{ inches}} \quad \text{Convert 8 feet to inches.}$$

$$= \frac{1}{96} \quad \text{Cancel the units.}$$

The scale factor is $\frac{1}{96}$. That is, each measure on the blueprint is $\frac{1}{96}$ the actual measure.

Your Turn

a. On a scale drawing of a new classroom, the scale is 1 centimeter = 2.5 meters. What is the scale factor?

STUDY TIP

Equivalent Scales

The scales below are equivalent because their scale factors are equal, $\frac{1}{96}$.

- 1 inch = 8 feet
- $\frac{1}{4}$ inch = 2 feet



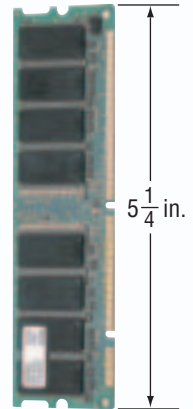
If you know the actual length of an object and the scale, you can build a scale model. A **scale model** can be used to represent something that is too large or too small for an actual-size model.

STUDY TIP

Scale The scale is the ratio of the drawing/model measure to the actual measure. It is not always the ratio of a smaller measure to a larger measure.

EXAMPLE Make a Scale Model

COMPUTERS Designers are creating a larger model of the computer memory board to use in design work. If they use a scale of 20 inches = 1 inch, what is the length of the model?



Write a proportion using the scale.

	Scale	=	Length of Model	
model →	$\frac{20 \text{ inches}}{1 \text{ inch}}$		$\frac{m \text{ inches}}{5\frac{1}{4} \text{ inches}}$	← model
actual →				← actual

$20 \cdot 5\frac{1}{4} = 1 \cdot m$ Find the cross products.

$105 = m$ Multiply.

The scale model is 105 inches long.

Skill and Concept Check

- OPEN ENDED** On grid paper, make a scale drawing of a room in your home. Include the scale that you used.
- NUMBER SENSE** Roberto built a model racecar that is 3 inches long and has a scale of 0.5 inch = 2 feet. He determined that the actual racecar must be $3 \cdot 2$ or 6 feet long. Is this correct? Explain.

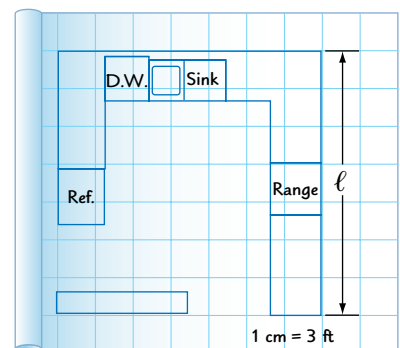
GUIDED PRACTICE

Find the actual distance between each pair of cities. Round to the nearest tenth if necessary.

	Cities	Map Distance	Scale
3.	Kokomo, Indiana, and Chicago, Illinois	8 cm	1 cm = 25 km
4.	Raleigh and Charlotte, North Carolina	$3\frac{1}{4}$ in.	1 in. = 40 mi

BLUEPRINTS For Exercises 5–7, use the blueprint at the right. Each square has a side length of 0.5 centimeter.

- What is the length ℓ of the kitchen on the blueprint?
- Find the actual length of the kitchen.
- Find the scale factor. (*Hint*: 1 ft \approx 30 cm)
- BRIDGES** A bridge is 28 meters long. Find the length of a scale model if the scale is 1 centimeter = 5.5 meters. Round to the nearest tenth.



Practice and Applications

Find the actual distance between each pair of cities. Round to the nearest tenth if necessary.

	Cities	Map Distance	Scale
9.	Virginia Beach, Virginia, and Washington, D.C.	8.5 cm	1 cm = 30 km
10.	Mobile and Huntsville, Alabama	$14\frac{1}{2}$ in.	1 in. = 20 mi
11.	Knoxville, Tennessee, and Choctaw, Oklahoma	19.28 cm	2 cm = 125 km
12.	Quebec, Canada, and Paris, France	$4\frac{5}{8}$ in.	$\frac{1}{2}$ in. = 355 mi

HOMWORK HELP

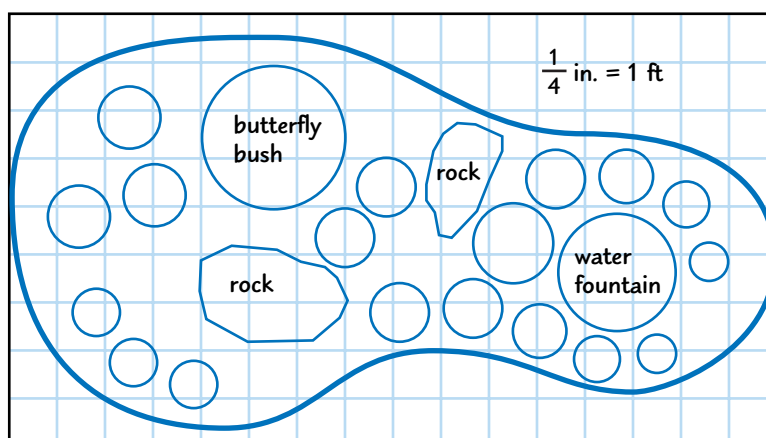
For Exercises	See Examples
9–12, 23	1
17–20	2
13–16, 21, 25	3
22, 24, 26	4

Extra Practice
See pages 581, 602.

Suppose you are making a scale drawing. Find the length of each object on the scale drawing with the given scale. Then find the scale factor.

- a back yard 120 feet deep; 1 inch : 20 feet
- an airplane 87 feet long; 2 inches = 15 feet
- an amusement park ride 36 meters high; 0.5 centimeter = 1.5 meters
- a surgical instrument $5\frac{7}{8}$ inches long; 1 inch : $\frac{1}{2}$ inch

For Exercises 17–21, refer to the blueprint of the garden shown below. Each square has a side length of $\frac{1}{4}$ inch.



- What is the length of the garden on the blueprint?
- Find the actual length of the garden.
- What is the diameter of the butterfly bush on the blueprint?
- What is the actual circumference of the butterfly bush? Use 3.14 for π and round to the nearest tenth.
- Find the scale factor.
- BRIDGES** The Natchez Trace Bridge in Franklin, Tennessee, is 1,500 feet long. Suppose you build a model of the bridge using a scale of 1 inch : 25 feet. How long is the model?



23. **GEOGRAPHY** A map of Brasilia has a scale of 1 inch to 5 miles. If the city is $2\frac{7}{16}$ inches across on the map, what is the distance across the actual city? Use estimation to check your answer.

ART For Exercises 24 and 25, refer to the figure at the right and the information below.

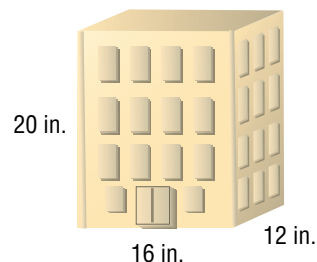
Mount Rushmore is a sculpture that was carved using a model with a scale of 1 inch : 1 foot, as shown at the right.



24. How high is George Washington's face on the actual sculpture of Mount Rushmore?
25. What is the scale factor?
26. **MODEL** The diameter of an igloo is 10 feet. If you were to make a model of the igloo with sugar cubes, which might be an appropriate scale: 1 inch = 1 foot or 1 inch = 50 feet? Explain.
27. **CRITICAL THINKING** The distance between Atlanta and Savannah, Georgia, is 245 miles. The distance on a map is $3\frac{1}{2}$ inches. What is the scale of the map?

Spiral Review with Standardized Test Practice

28. **SHORT RESPONSE** A scale model of a new office building is shown at the right. If the scale is 5 inches = 18 feet, how tall is the actual building?



29. **MULTIPLE CHOICE** Which of the following scales does *not* have a scale factor of $\frac{1}{60}$?

- (A) 1 in. = 5 ft (B) 1 cm = 60 cm
(C) 1 ft = 20 yd (D) 1 in. = 60 ft

30. **BIRDS** A binocular has a *magnification factor* of 35 to 1; that is, objects that are actually 35 feet away appear to be only 1 foot away using the binoculars. If a goldfinch is 368 feet away, what will the distance appear to be in the binoculars? Round to the nearest tenth. (Lesson 7-3)

Find each unit rate. (Lesson 7-2)

31. 200 miles in 5 hours 32. 99¢ for 30 ounces
33. \$6.20 for 4 pounds 34. 150 meters in 12 seconds

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Divide. Write in simplest form. (Lesson 6-6)

35. $2\frac{3}{4} \div 10$ 36. $4\frac{1}{3} \div 10$ 37. $30\frac{2}{3} \div 100$ 38. $87\frac{1}{2} \div 100$

What You'll LEARN

Use a spreadsheet to calculate measurements for scale drawings.

Scale Drawings

A computer spreadsheet is a useful tool for calculating measures for scale drawings. You can change the scale factors and the dimensions, and the spreadsheet will automatically calculate the new values.

ACTIVITY

Suppose you want to make a scale drawing of your school. Set up a spreadsheet like the one shown below. In this spreadsheet, the actual measures are in feet, and the scale drawing measures are in inches.

	A	B	C
	Actual Measure (ft)	Scale Factor	Scale Drawing Measure (in.)
1			
2	5	60	1.0
3	30	100	3.6
4	100	250	4.8
5			
6			
7			

Formula bar: $= (A3 * 12) / B3$

Callout 1: Multiply each actual measure by 12 to convert to inches. Then divide by the scale factor.

Callout 2: The scale factor 100 means that the ratio of drawing measures to actual measures is 1:100.

EXERCISES

- The length of one side of the school building is 100 feet. If you use a scale factor of 1:250, what is the length on your scale drawing?
- The length of a classroom is 30 feet. What is the scale factor if the length of the classroom on a scale drawing is 3.6 inches?
- Calculate the length of a 30-foot classroom on a scale drawing if the scale factor is 1:10.
- Suppose the actual measures of your school are given in meters. Describe how you could use a spreadsheet to calculate the scale drawing measures in centimeters using a scale factor of 1:50.
- Choose three rooms in your home and use a spreadsheet to make scale drawings. First, choose an appropriate scale and calculate the scale factor. Include a sketch of the furniture in each room.

Mid-Chapter Practice Test

Vocabulary and Concepts

1. **Explain** the difference between a rate and a unit rate and give an example of each. (Lesson 7-2)
2. **Define** cross product. (Lesson 7-3)

Skills and Applications

Write each ratio as a fraction in simplest form. (Lesson 7-1)

3. 18 to 12
4. 7 to 49
5. 20 meters : 44 meters

Determine whether the ratios are equivalent. Explain. (Lesson 7-1)

6. $\frac{6}{9}$ and $\frac{2}{3}$
7. 150 : 15 and 3 : 1
8. 24 : 4 and 72 : 8

Find each unit rate. Round to the nearest hundredth if necessary. (Lesson 7-2)

9. 200 miles in 5 hours
10. \$6.20 for 5 pounds

11. **SHOPPING** Which box of cereal shown in the table costs the least per ounce? (Lesson 7-2)

Weight (oz)	Cost (\$)
12	2.50
18	3.69
24	4.95

12. Determine whether the ratios $\frac{5}{8}$ and $\frac{25}{40}$ form a proportion. (Lesson 7-3)

Solve each proportion. (Lesson 7-3)

13. $\frac{3}{d} = \frac{12}{20}$
14. $\frac{7}{8} = \frac{m}{48}$
15. $\frac{w}{8} = \frac{1}{3}$

16. **TRUCKS** A truck is 26 feet long. Find the length on a scale model if 1 inch represents $5\frac{1}{2}$ feet. Round to the nearest tenth if necessary. (Lesson 7-4)

Suppose you are making a scale drawing. Find the length of each object on the scale drawing with the given scale. Then find the scale factor. (Lesson 7-4)

17. a bedroom 12 feet long; 1 in. = 3 ft
18. a park area 38 meters wide; 0.5 cm = 1 m

Standardized Test Practice

19. **GRID IN** A train travels 146 miles in 2 hours. At this rate, how many miles will it travel in 3.5 hours? (Lesson 7-3)
20. **SHORT RESPONSE** On a scale drawing, a dog pen is $4\frac{1}{2}$ inches long. If 1 inch = 3 feet, what is the actual length of the dog pen? (Lesson 7-4)

The Game Zone

A Place To Practice Your Math Skills

Math Skill

Solving Proportions



Racing with Proportions

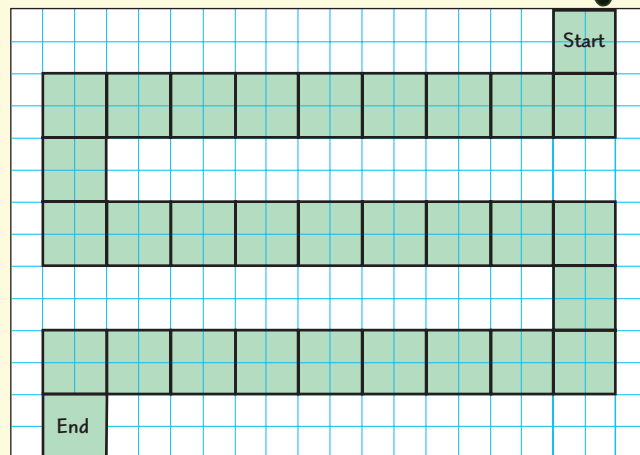
● GET READY!

Players: two

Materials: centimeter grid paper, spinner, centimeter cubes

● GET SET!

- Copy the game board at the right onto grid paper.



- Label equal sections of a spinner with 10, 12, 15, 20, 24, 30, 40, and 60.



● GO!

- Each player places a cube on the *Start* square. (Note that both cubes will not fit on the same square. Place them side by side.)
- Each player spins the spinner and substitutes the number on the spinner for x in the proportion $\frac{y}{15} = \frac{8}{x}$. The player solves the proportion and moves his or her centimeter cube y spaces.
- Then each player spins the spinner and substitutes the number on the spinner for y in the given proportion. He or she solves the proportion and moves his or her centimeter cube x spaces.
- Continue to spin the spinner and solve the proportion for y and x .
- **Who Wins?** The first person to reach the *End* square wins the round. It is not necessary to land on *End* with an exact roll.

Fractions, Decimals, and Percents



What You'll LEARN

Write percents as fractions, and vice versa.

REVIEW Vocabulary

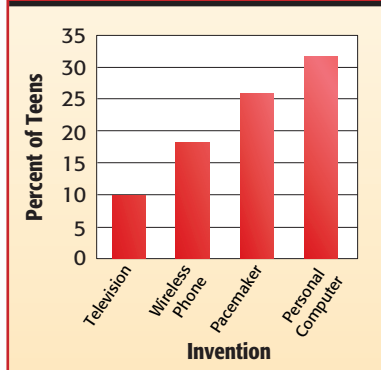
percent: a ratio that compares a number to 100 (Lesson 5-5)

WHEN am I ever going to use this?

SURVEYS The graph shows the results of a survey in which teens were asked to name the most important invention of the 20th century.

1. What percent of the teens said that the personal computer was the most important invention?
2. How is this percent written as a ratio?
3. Simplify the ratio.

Most Important Inventions



Source: Lemelson-MIT Program

In Lesson 5-5, you wrote percents like 32% as fractions by writing fractions with denominators of 100 and then simplifying. You can use the same method to write percents like 16.8% and $8\frac{1}{3}\%$ as fractions.

EXAMPLES

Percents as Fractions

- HOCKEY** In men's college hockey, 16.8% of the players are from Ontario, Canada. What fraction is this? Write in simplest form.

$$16.8\% = \frac{16.8}{100}$$

Write a fraction with a denominator of 100.

$$= \frac{16.8}{100} \cdot \frac{10}{10}$$

Multiply by $\frac{10}{10}$ to eliminate the decimal in the numerator.

$$= \frac{168}{1,000} \text{ or } \frac{21}{125}$$

Simplify.

So, $\frac{21}{125}$ of the players are from Ontario, Canada.

- Write $8\frac{1}{3}\%$ as a fraction in simplest form.**

$$8\frac{1}{3}\% = \frac{8\frac{1}{3}}{100}$$

Write a fraction.

$$= 8\frac{1}{3} \div 100$$

Divide.

$$= \frac{25}{3} \div 100$$

Write $8\frac{1}{3}$ as an improper fraction.

$$= \frac{25}{3} \times \frac{1}{100}$$

Multiply by the reciprocal of 100, which is $\frac{1}{100}$.

$$= \frac{25}{300} \text{ or } \frac{1}{12}$$

Simplify.

To write a fraction like $\frac{8}{25}$ as a percent, multiply the numerator and the denominator by a number so that the denominator is 100. If the denominator is not a factor of 100, you can write fractions as percents by using a proportion.

EXAMPLES Fractions as Percents

TESTS On a math test, Mary got 7 questions correct out of 8. Find her grade as a percent.

To find her grade, write $\frac{7}{8}$ as a percent. **Estimate** $\frac{7}{8}$ is greater than $\frac{6}{8} = \frac{3}{4}$, or greater than 75%.

$$\frac{7}{8} = \frac{n}{100} \quad \text{Write a proportion using } \frac{n}{100}.$$

$$700 = 8n \quad \text{Find the cross products.}$$

$$\frac{700}{8} = \frac{8n}{8} \quad \text{Divide each side by 8.}$$

$$87\frac{1}{2} = n \quad \text{Simplify.}$$

$$\text{So, } \frac{7}{8} = 87\frac{1}{2}\% \text{ or } 87.5\%.$$

This is greater than 75%, which was the estimate.

Write $\frac{4}{15}$ as a percent. Round to the nearest hundredth.

Estimate $\frac{4}{15}$ is about $\frac{4}{16}$, which equals $\frac{1}{4}$ or 25%.

$$\frac{4}{15} = \frac{n}{100} \quad \text{Write a proportion using } \frac{n}{100}.$$

$$400 = 15n \quad \text{Find the cross products.}$$

$$400 \div 15 = 26.6666667 \quad \text{Use a calculator to simplify.}$$

So, $\frac{4}{15}$ is about 26.67%. This result is close to the estimate.

Your Turn Write each fraction as a percent. Round to the nearest hundredth if necessary.

a. $\frac{2}{15}$

b. $\frac{7}{16}$

c. $\frac{17}{24}$

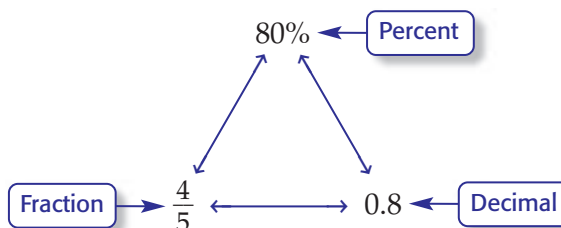
STUDY TIP

Choose the Method

To write a fraction as a percent,

- use *multiplication* when a fraction has a denominator that is a factor of 100,
- use a *proportion* for any type of fraction.

In this lesson, you have written percents as fractions and fractions as percents. In Chapter 5, you wrote percents and fractions as decimals. You can also write a fraction as a percent by first writing the fraction as a decimal and then writing the decimal as a percent.



Percents, fractions, and decimals are all different names that represent the same number.



STUDY TIP

Look Back You can review writing fractions as decimals in Lesson 5-4.

EXAMPLES

Fractions as Percents

- 5 Write $\frac{1}{8}$ as a percent.

$$\begin{aligned}\frac{1}{8} &= 0.125 && \text{Write } \frac{1}{8} \text{ as a decimal.} \\ &= 12.5\% && \text{Multiply by 100 and add the \%}.\end{aligned}$$

- 6 Write $\frac{5}{6}$ as a percent. Round to the nearest hundredth.

$$\begin{aligned}\frac{5}{6} &= 0.833333333\dots && \text{Write } \frac{5}{6} \text{ as a decimal.} \\ &\approx 83.33\% && \text{Multiply by 100 and add the \%}.\end{aligned}$$

- 7 **Your Turn** Write each fraction as a percent. Round to the nearest hundredth if necessary.

d. $\frac{5}{16}$

e. $\frac{7}{12}$

f. $\frac{2}{9}$

Some fractions whose denominators are not factors of 100 are used often in everyday situations. It is helpful to be familiar with these fractions and their equivalent decimals and percents.

Noteables

Key Concept: Common Equivalents

Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}\%$	$\frac{3}{8}$	0.375	$37\frac{1}{2}\%$
$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}\%$	$\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$	$\frac{7}{8}$	0.875	$87\frac{1}{2}\%$

Skill and Concept Check

- Writing Math** Describe two methods for writing a fraction as a percent.
- OPEN ENDED** Write a percent that is between 15.2% and 15.8%. Then write it as a fraction in simplest form.
- Explain** why percents are used in statistical graphs rather than fractions.

GUIDED PRACTICE

Write each percent as a fraction in simplest form.

4. 13.5% 5. 18.75% 6. $66\frac{2}{3}\%$ 7. $57\frac{1}{2}\%$

Write each fraction as a percent. Round to the nearest hundredth if necessary.

8. $\frac{4}{25}$ 9. $\frac{5}{8}$ 10. $\frac{9}{40}$ 11. $\frac{4}{11}$

12. **LANGUAGES** In Virginia, 1 person out of every 40 speaks an Asian language at home. What percent is this?

Practice and Applications

HOMWORK HELP

For Exercises 13–24, 38 25–37, 39	See Examples 1, 2 3–6
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Extra Practice
See pages 581, 602.

Write each percent as a fraction in simplest form.

13. 17.5% 14. 62.5% 15. 6.2% 16. 34.5%
 17. 28.75% 18. 56.25% 19. $33\frac{1}{3}\%$ 20. $16\frac{2}{3}\%$
 21. $81\frac{1}{4}\%$ 22. $93\frac{3}{4}\%$ 23. $78\frac{3}{4}\%$ 24. $23\frac{1}{3}\%$

Write each fraction as a percent. Round to the nearest hundredth if necessary.

25. $\frac{11}{20}$ 26. $\frac{18}{25}$ 27. $\frac{3}{8}$ 28. $\frac{21}{40}$ 29. $\frac{29}{30}$ 30. $\frac{8}{9}$
 31. $\frac{5}{7}$ 32. $\frac{1}{16}$ 33. $\frac{1}{80}$ 34. $\frac{57}{200}$ 35. $\frac{5}{12}$ 36. $\frac{7}{15}$
 37. What percent is equivalent to $\frac{60}{125}$?

CARS For Exercises 38 and 39, use the table at the right. It shows the percent of people who keep the listed items in their car.

38. What fraction of people keep a first-aid kit in their car?
 39. Approximately 26 out of 125 people surveyed keep a hairbrush in their car. Is this greater or less than the percent who keep sports equipment? Explain.
 40. **CRITICAL THINKING** For what value of x is $\frac{1}{x} = x\%$ a true sentence?

Items in Car	Percent of People
pen/pencil	73.0%
cassette tapes/CDs	66.1%
first-aid kit	38.2%
sports equipment	28.9%

Source: Penzoil "How's your Automotive Attitude?" survey

Spiral Review with Standardized Test Practice

41. **MULTIPLE CHOICE** Two-thirds of Kodiak Island's 2 million acres has been set aside as a wildlife preserve for the Kodiak brown bear. What percent of the island is a preserve?
 Ⓐ $2\frac{2}{3}\%$ Ⓑ 23% Ⓒ $33\frac{1}{3}\%$ Ⓓ $66\frac{2}{3}\%$
 42. **SHORT RESPONSE** What is $\frac{7}{16}$ written as a percent?
 43. **MODELS** On a scale model of a building, 2 inches represents 15 feet. If the model is 9 inches tall, how tall is the actual building? (Lesson 7-4)

Solve each proportion. (Lesson 7-3)

44. $\frac{6}{9} = \frac{4}{x}$ 45. $\frac{3}{t} = \frac{9}{5}$ 46. $\frac{n}{5} = \frac{2.8}{7}$ 47. $\frac{7}{12} = \frac{r}{30}$

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Write each decimal as a fraction with a denominator of 100. (Lesson 5-4)

48. 0.23 49. 0.09 50. 0.368 51. 0.425



Percents Greater Than 100% and Percents Less Than 1%

What You'll LEARN

Write percents greater than 100% and percents less than 1% as fractions and as decimals, and vice versa.

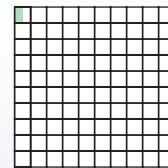
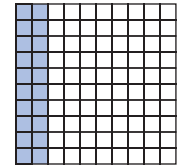
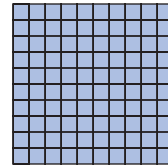
HANDS-ON Mini Lab

Materials

- grid paper
- colored pencils

Work with a partner.

- Draw three 10×10 squares on a piece of grid paper. Each large square represents 100%, and each small square represents 1%.
- For the first model, use two grids to shade 120 small squares.
- For the second model, shade half of one small square on a grid. Use a different color than on the first model.



1. Which model represents a percent greater than 100%? What is the percent?
2. Which model represents a percent less than 1%? What is the percent?
3. Shade grids to represent each percent.
 - a. 150%
 - b. 215%
 - c. $\frac{1}{4}\%$

Percents greater than 100% or less than 1% can be written as decimals using the methods you learned in Chapter 5. They can also be written as mixed numbers or fractions.

$$\begin{aligned} 250\% &= \frac{250}{100} \\ &= 2.5 \text{ or } 2\frac{1}{2} \end{aligned}$$

A percent greater than 100% equals a number greater than 1.

$$\begin{aligned} 0.5\% &= \frac{0.5}{100} \\ &= 0.005 \text{ or } \frac{1}{200} \end{aligned}$$

A percent less than 1% equals a number less than 0.01 or $\frac{1}{100}$.

STUDY TIP

Alternate Method

To divide by 100, move the decimal point two places to the left.

$$410\% = \underline{4}10 \text{ or } 4.1$$

$$0.2\% = \underline{00}2 \text{ or } 0.002$$

EXAMPLES

Percents as Decimals or Fractions

Write each percent as a decimal and as a mixed number or fraction in simplest form.

1 410%

$$\begin{aligned} 410\% &= \frac{410}{100} && \text{Definition of percent} \\ &= 4.1 \text{ or } 4\frac{1}{10} \end{aligned}$$

2 0.2%

$$\begin{aligned} 0.2\% &= \frac{0.2}{100} && \text{Definition of percent} \\ &= 0.002 \text{ or } \frac{1}{500} \end{aligned}$$

READING Math

Percents In the example at the right, the solution 3 means *3 times as much*. So, before hibernation bears need 3 times as many Calories as usual.

EXAMPLE

Real-Life Percents as Decimals

- BEARS** Before bears hibernate, their need for Calories increases 300%. Write 300% as a decimal.

$$\begin{aligned} 300\% &= \frac{300}{100} \quad \text{Divide by 100.} \\ &= 3.0 \text{ or } 3 \end{aligned}$$



You can write decimals less than 0.01 and decimals greater than 1 as percents by multiplying by 100 and adding a %.

EXAMPLES

Decimals as Percents

Write each decimal as a percent.

- 1** 1.68

$$\begin{aligned} 1.68 &= \frac{1.68}{1} \quad \text{Multiply by 100.} \\ &= 168\% \end{aligned}$$

- 5** 0.0075

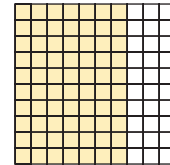
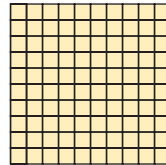
$$\begin{aligned} 0.0075 &= \frac{0.0075}{1} \quad \text{Multiply by 100.} \\ &= 0.75\% \end{aligned}$$

- Your Turn** Write each decimal as a percent.

- a. 4.5 b. 0.002 c. 0.0016

Skill and Concept Check

- Write the fraction of squares that are shaded as a percent and as a decimal.
- OPEN ENDED** Write a number between 6 and 7 as a percent.
- Which One Doesn't Belong?** Identify the number that does not have the same value as the other three. Explain your reasoning.



63.5%

0.635

$63\frac{1}{2}$

$63\frac{1}{2}\%$

GUIDED PRACTICE

Write each percent as a decimal and as a mixed number or fraction in simplest form.

4. 325% 5. 200% 6. 0.7% 7. 0.15%

Write each decimal as a percent.

8. 1.8 9. 0.0015 10. 2.75 11. 0.0005

12. **PHOTOGRAPHY** A photograph developed from 35 mm film is 428% larger than the negative. Express this percent as a decimal.

Practice and Applications

Write each percent as a decimal and as a mixed number or fraction in simplest form.

13. 350% 14. 475% 15. 600%
16. 400% 17. 0.6% 18. 0.1%
19. 0.55% 20. 0.48% 21. 260%
22. 195% 23. 0.05% 24. 0.04%

Write each decimal as a percent.

25. 8.5 26. 35 27. 0.009 28. 0.003
29. 2.64 30. 1.07 31. 0.0034 32. 0.0077

Write each number as a percent.

33. $3\frac{1}{2}$ 34. $9\frac{3}{4}$ 35. $\frac{2}{500}$ 36. $\frac{1}{400}$
37. Write $1\frac{5}{8}$ as a percent. 38. Write $\frac{1}{8}\%$ as a decimal.

39. **POPULATION** In 2000, about 0.3% of the population in the United States was Japanese. Write this percent as a decimal and as a fraction.
40. **RESEARCH** Use the Internet or another source to find the percent of people in the United States that come from three other countries. Write the percents as decimals.
41. **MULTI STEP** One hour is what percent of one week?
42. **CRITICAL THINKING** Some North African ostriches are 150% of the height of a 6-foot human. What is the height of some ostriches?

Spiral Review with Standardized Test Practice

43. **MULTIPLE CHOICE** A certain stock increased its value by 467% over 10 years. What is this percent written as a decimal?
 A 4.67 B 46.7 C 0.467 D 0.00467
44. **MULTIPLE CHOICE** Choose the best estimate for 100.5%.
 F 0 G 1 H 100 I 1,000

Write each percent as a fraction in simplest form. (Lesson 7-5)

45. 7.5% 46. 1.2% 47. $6\frac{1}{4}\%$ 48. $92\frac{1}{2}\%$
49. **SCALE DRAWING** A garage is 18 feet wide. In a scale drawing, 1 inch = 3 feet. What is the width of the garage on the drawing? (Lesson 7-4)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Write each percent as a decimal. (Lesson 5-6)

50. 85% 51. 6.5% 52. 36.9% 53. 12.3%

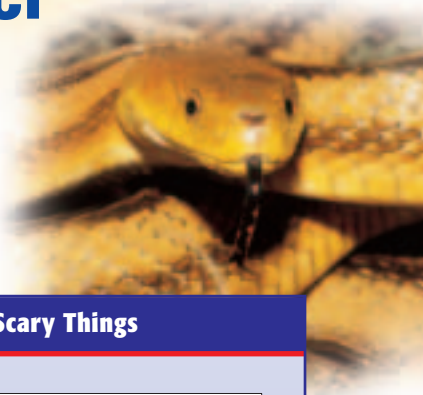
HOMEWORK HELP

For Exercises	See Examples
13–24, 33–37	1, 2
39–40	3
25–32, 38	4, 5

Extra Practice
See pages 581, 602.

7-7

Percent of a Number



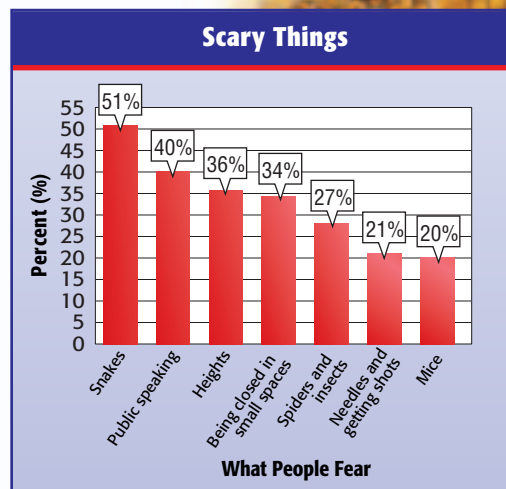
What You'll LEARN

Find the percent of a number.

WHEN am I ever going to use this?

SURVEYS The graph at the right shows the results of a survey in which 1,016 people were asked to name the things they feared.

1. About how many of them said they were afraid of snakes?
2. How did you estimate?
3. Suppose you pass 50 people at a mall. Based on the results of the survey, do you think more or less than 25 of them are afraid of mice? Explain.



Source: Gallup Poll

One way to find the percent of a number is to use a proportion.

EXAMPLE Use a Proportion to Find a Percent

SURVEYS The graphic above shows that 36% of 1,016 adults surveyed were afraid of heights. How many of the people surveyed were afraid of heights?

36% means that 36 out of 100 people were afraid of heights. Find an equivalent ratio x out of 1,016 and write a proportion.

$$\left. \begin{array}{l} \text{number afraid of heights} \rightarrow \frac{x}{1,016} \\ \text{total number in survey} \rightarrow \frac{36}{100} \end{array} \right\} \begin{array}{l} \text{percent of people} \\ \text{afraid of heights} \end{array}$$

Now solve the proportion.

$$\frac{x}{1,016} = \frac{36}{100} \quad \text{Write the proportion.}$$

$$x \cdot 100 = 1,016(36) \quad \text{Find the cross products.}$$

$$100x = 36,576 \quad \text{Multiply.}$$

$$\frac{100x}{100} = \frac{36,576}{100} \quad \text{Divide each side by 100.}$$

$$x = 365.76 \quad \text{Simplify.}$$

So, about 366 of the 1,016 people surveyed were afraid of heights.



Another method for finding the percent of a number is to use multiplication.

EXAMPLES

Use Multiplication to Find a Percent

1 What number is 140% of 32?

$$140\% \text{ of } 32 = 140\% \times 32 \quad \text{Write a multiplication expression.}$$

$$= 1.4 \times 32 \quad \text{Write 140\% as a decimal.}$$

$$= 44.8 \quad \text{Multiply.}$$

So, 140% of 32 is 44.8.

2 Find 75% of \$800.

$$75\% \text{ of } \$800 = 75\% \times \$800 \quad \text{Write a multiplication expression.}$$

$$= 0.75 \times 800 \quad \text{Write 75\% as a decimal.}$$

$$= 600 \quad \text{Multiply.}$$

So, 75% of \$800 is \$600.

3 Your Turn Find each number.

a. Fifty percent of 140 is what number?

b. Find 37.5% of 64.

STUDY TIP

Mental Math You can find 75% of 800 mentally by using the fraction $\frac{3}{4}$. $\frac{3}{4}$ of 800 is 600.

Skill and Concept Check

1. Write a proportion that can be used to find 0.5% of 22.
2. **NUMBER SENSE** Use mental math to determine whether 30% of \$150 is \$80. Explain.
3. **OPEN ENDED** Write a problem that can be solved using the graphic at the right. Then solve the problem.

GUIDED PRACTICE

Find each number. Round to the nearest tenth if necessary.

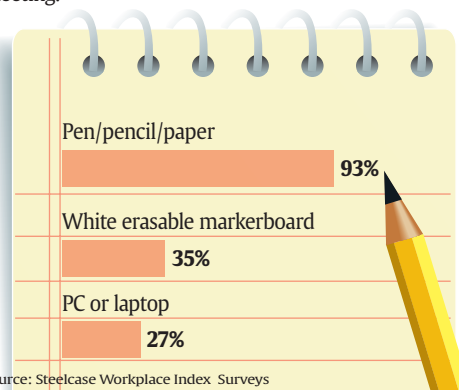
4. Find 95% of \$40.
5. Forty-two percent of 263 is what number?
6. What number is 200% of 75?
7. Find 0.25% of 40.
8. **COMMISSION** Ms. Sierra earns a 2% *commission*, or fee paid based on a percent of her sales, on every vacation package that she sells. One day, she sold vacation packages worth \$6,500. What was her commission?



USA TODAY Snapshots®

Low-tech tools rule at meetings

1,500 white-collar workers were asked what tools they most commonly used to record information during a meeting.



Source: Steelcase Workplace Index Surveys

By Darryl Haralson and Keith Simmons, USA TODAY

Practice and Applications

HOMWORK HELP

For Exercises 9–23 See Examples

1–3

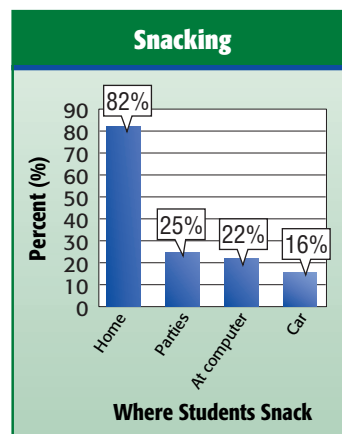
Extra Practice
See pages 582, 602.

Find each number. Round to the nearest tenth if necessary.

9. What is 65% of 100?
10. Find 45% of \$400.
11. Find 25% of \$640.
12. What number is 18.5% of 500?
13. 67.5% of 76 is what?
14. 64% of 88 is what number?
15. What is 125% of 10?
16. Find 300% of 14.
17. Find 0.15% of 250
18. What number is 0.5% of 80?
19. Find $20\frac{1}{4}\%$ of 3.
20. 250.5% of 25 is what number?

SNACKING For Exercises 21 and 22, use the graphic at the right. It shows the results of a poll of 1,746 college students.

21. Write a proportion that can be used to find how many students like to snack at their computer. Then solve.
22. How many students surveyed like to snack at home?
23. **DELIVERY** A mountain bike costs \$288 plus a 4.5% delivery charge. What is the cost of the bike including the delivery charge?



Source: Snack Food Association

24. **CRITICAL THINKING** Suppose you add 10% of a number to the number. Then you subtract 10% of the total. Is the result *greater than*, *less than*, or *equal to* the original number? Explain your reasoning.

Spiral Review with Standardized Test Practice

25. **MULTIPLE CHOICE** A football player has made 80% of the field goals he has attempted in his career. If he attempts 5 field goals in a game, how many would he be expected to make?
 A 5 B 4 C 3 D 2

26. **SHORT RESPONSE** What number is 12% of 16.5?

Write each decimal as a percent. (Lesson 7-6)

27. 7.5 28. 9 29. 0.0004 30. 0.0018
31. Write $\frac{11}{15}$ as a percent. Round to the nearest hundredth. (Lesson 7-5)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Evaluate each expression. Round to the nearest tenth if necessary. (Lesson 1-3)

32. $\frac{36 \cdot 2}{80}$
33. $\frac{14(5)}{3.5}$
34. $\frac{9 \times 100}{56}$
35. $22 \cdot 100 \div 490$



What You'll LEARN

Use a percent model to find a part.

Materials

- grid paper

Using a Percent Model

You can find the percent of a number or the *part* of the whole by using a model.

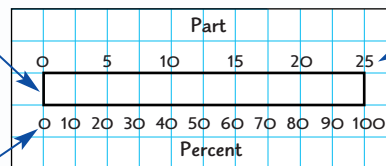
ACTIVITY

Work with a partner.

Suppose you and a friend eat dinner at the Pizza Palace and your total bill is about \$25. If you want to leave a 20% tip, how much should you leave? Use a model to find the amount.

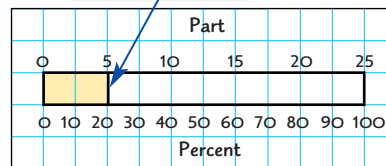
Draw a 10-unit by 1-unit rectangle on grid paper.

Label equal units from 0 to 100 to represent percents.



Label equal units from 0 to 25 to represent parts of the whole quantity, 25.

Draw a vertical line at 20%.



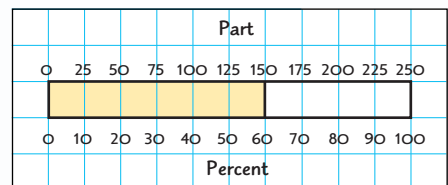
So, you should leave \$5 as a tip.

Your Turn Draw a model to find each part.

- a. 70% of 50 b. 45% of 20 c. 20% of 75

Writing Math

- Suppose your whole family had dinner with you and the total bill was \$50. How could you change your model to find the tip?
- Write** a sentence that describes what is represented by the model at the right.
- Write** a percent problem that can be represented and solved using a model.
- Make a conjecture** about how you could use a model to estimate the percent represented by the ratio 8 out of 50.



7-8

The Percent Proportion

What You'll LEARN

Solve problems using the percent proportion.

NEW Vocabulary

percent proportion
part
base

WHEN am I ever going to use this?

SPACE The engine on the space shuttle weighs approximately 19,700 pounds. The entire space shuttle weighs 178,000 pounds.

1. Write the ratio of engine weight to total weight as a fraction.
2. Use a calculator to write the fraction as a decimal to the nearest hundredth.
3. What percent of the space shuttle's weight is the engine?



A **percent proportion** compares **part** of a quantity to the whole quantity, called the **base**, using a percent. You can use a percent proportion to find what percent of a space shuttle's weight is the engine.

$$\begin{array}{l} \text{part: engine weight} \rightarrow \frac{19,700}{178,000} = \frac{p}{100} \\ \text{base: total weight} \rightarrow \end{array}$$

The percent is written as a fraction with a denominator of 100.

Noteables

Key Concept: Percent Proportion

Words The percent proportion is $\frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100}$.

Symbols $\frac{a}{b} = \frac{p}{100}$, where a is the part, b is the base, and p is the percent.

To find the percent, write the percent proportion and solve for p .

EXAMPLE Find the Percent

I What percent of \$15 is \$9?

9 is the part, and 15 is the base. You need to find the percent p .

$$\frac{a}{b} = \frac{p}{100}$$

Percent proportion

$$\frac{9}{15} = \frac{p}{100}$$

Replace a with 9 and b with 15.

$$9 \cdot 100 = 15 \cdot p$$

Find the cross products.

$$900 = 15p$$

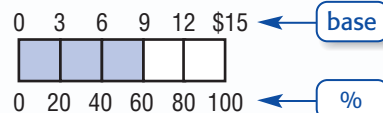
Simplify.

$$\frac{900}{15} = \frac{15p}{15}$$

Divide each side by 15.

$$60 = p$$

So, 60% of \$15 is \$9. Compare the answer to the model.



STUDY TIP

Finding the Base In a percent problem, the base follows the word *of*.

You can also use the percent proportion to find a missing part or base.

Concept Summary

Types of Percent Problems

Type	Example	Proportion
Find the Percent	What percent of 6 is 3?	$\frac{3}{6} = \frac{p}{100}$
Find the Part	What number is 50% of 6?	$\frac{a}{6} = \frac{50}{100}$
Find the Base	3 is 50% of what number?	$\frac{3}{b} = \frac{50}{100}$

To find the part, write the percent proportion and solve for a . To find the base, write the percent proportion and solve for b .

EXAMPLE Find the Part

1 What number is 40% of 120?

40 is the percent and 120 is the base. You need to find the part.

$$\frac{a}{b} = \frac{p}{100}$$

Percent proportion

$$\frac{a}{120} = \frac{40}{100}$$

Replace b with 120 and p with 40.

$$a \cdot 100 = 120 \cdot 40$$

Find the cross products.

$$100a = 4,800$$

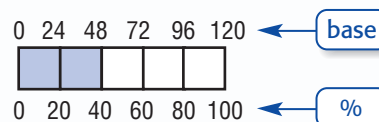
Simplify.

$$\frac{100a}{100} = \frac{4,800}{100}$$

Divide each side by 100.

$$a = 48$$

So, 40% of 120 is 48. Compare to the model.



EXAMPLE Find the Base

1 18 is 25% of what number?

18 is the part and 25 is the percent. You need to find the base, or the whole quantity.

$$\frac{a}{b} = \frac{p}{100}$$

Percent proportion

$$\frac{18}{b} = \frac{25}{100}$$

Replace a with 18 and p with 25.

$$18 \cdot 100 = b \cdot 25$$

Find the cross products.

$$1,800 = 25b$$

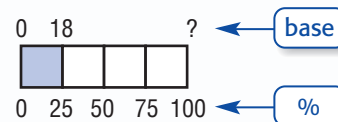
Simplify.

$$\frac{1,800}{25} = \frac{25b}{25}$$

Divide each side by 25.

$$72 = b$$

So, 18 is 25% of 72.



Check In the model above, the whole quantity is $18 \cdot 4$ or 72.

Your Turn Find each number. Round to the nearest tenth if necessary.

- a. What number is 5% of 60? b. 80 is 75% of what number?

Skill and Concept Check

- Choose the problem that can be solved by using $\frac{8}{b} = \frac{16}{100}$.
 - What number is 16% of 8?
 - 8 is 16% of what number?
- OPEN ENDED** Write a proportion that can be used to find the part if you know the base and the percent.

GUIDED PRACTICE

Find each number. Round to the nearest tenth if necessary.

- What percent of \$50 is \$18?
- What number is 25% of 180?
- What number is 2% of 35?
- 62 is 90.5% of what number?

Practice and Applications

Find each number. Round to the nearest tenth if necessary.

- What percent of 60 is 15?
 - 9 is 45% of what number?
 - What number is 12% of 72?
 - What percent of \$12 is \$9?
 - 75 is 20% of what number?
 - What number is 40% of 80?
 - \$3 is what percent of \$40?
 - 12.5% of what number is 24?
 - 8.2% of 50 is what number?
 - What percent of 300 is 0.6?
 - 45 is 150% of what number?
 - What percent of 25 is 30?
 - What number is 0.5% of 8?
 - 6% of what number is $10\frac{1}{2}$?
21. **SHOPPING** A paperback book originally priced at \$12.50 is on sale for \$7.50. What percent of the original cost is the sale price?
22. **MOVIES** Sarah and Monique spent \$14, or 35% of their money, on movie tickets. How much money did they have to start with?
23. **CRITICAL THINKING** Without calculating, order the following from greatest to least value. Explain your reasoning.

20% of 100, 20% of 500, 5% of 100

HOMESCHOOL HELP

For Exercises 7–22 See Examples 1–3

Extra Practice
See pages 582, 602.

Spiral Review with Standardized Test Practice

24. **MULTIPLE CHOICE** There are about 53,500 hotels in the United States. Use the table to find how many are located along highways.
- (A) 22,577 (B) 22,470 (C) 17,976 (D) 5,457
25. **SHORT RESPONSE** 95 of 273 students volunteered. About what percent of the students did *not* volunteer?

U.S. Hotels	
Location	Percent
highways	42.2%
suburbs	33.6%
city	10.2%

Find each number. Round to the nearest tenth if necessary. (Lesson 7-7)

- What is 25% of 120?
- Find 45% of 70.
- Write 1.2 as a percent. (Lesson 7-6)



Vocabulary and Concept Check

base (p. 323)

cross product (p. 297)

equivalent ratios (p. 289)

part (p. 323)

percent proportion (p. 323)

proportion (p. 297)

rate (p. 292)

scale (p. 304)

scale drawing (p. 304)

scale factor (p. 305)

scale model (p. 306)

unit rate (p. 292)

Choose the letter of the term that best matches each phrase.

- | | |
|---|---|
| <ol style="list-style-type: none"> 1. a comparison of two numbers by division 2. two ratios that have the same value 3. a ratio of two measurements with different units 4. an equation that shows that two ratios are equivalent 5. used to represent something that is too large or too small for an actual-size drawing 6. the ratio of the distance on a map to the actual distance 7. the number that is compared to the whole quantity in a percent proportion 8. product of the numerator of one ratio in a proportion and the denominator of the other ratio 9. a rate that is simplified so that it has a denominator of 1 10. the number that represents the whole quantity in a percent proportion | <ol style="list-style-type: none"> a. rate b. base c. scale drawing d. unit rate e. ratio f. scale g. part h. proportion i. equivalent ratios j. cross products |
|---|---|

Lesson-by-Lesson Exercises and Examples

7-1 Ratios (pp. 288–291)

Write each ratio as a fraction in simplest form.

11. 16:12 12. 5 to 25
13. 50 cm to 75 cm 14. 6 ft:12 in.

Determine whether the ratios are equivalent. Explain.

15. $\frac{3}{5}$ and $\frac{21}{35}$ 16. $\frac{18}{24}$ and $\frac{5}{20}$
17. 27:15 and 9:5 18. 4:21 and 2:7

Example 1 Write the ratio 32 to 18 as a fraction in simplest form.

$$\begin{aligned} 32 \text{ to } 18 &= \frac{32}{18} && \text{Write the ratio as a fraction.} \\ &= \frac{16}{9} && \text{Simplify.} \end{aligned}$$

Example 2 Determine whether 5:6 and 15:18 are equivalent.

$$5:6 = \frac{5}{6} \qquad 15:18 = \frac{15}{18} \text{ or } \frac{5}{6}$$

The ratios in simplest form both equal $\frac{5}{6}$.
So, 5:6 and 15:18 are equivalent.

7-2 Rates (pp. 292–295)

Find each unit rate. Round to the nearest hundredth if necessary.

19. \$23.75 for 5 pounds
20. 810 miles in 9 days
21. \$38 in 4 hours
22. 24 gerbils in 3 cages
23. 14 laps in 4 minutes

24. **SHOPPING** Which bottle of laundry detergent shown at the right costs the least per ounce?

Bottle	Price
12 oz	\$2.16
32 oz	\$5.60
64 oz	\$10.24

Example 3 Find the unit price per ounce of a 16-ounce box of pasta that is on sale for 96¢.

To find the cost per ounce, write the rate as a fraction and then simplify.

$$\begin{aligned} 16\text{-oz box for } 96\text{¢} &= \frac{96\text{¢}}{16 \text{ ounces}} \\ &= \frac{96\text{¢} \div 16}{16 \text{ ounces} \div 16} \\ &= \frac{6\text{¢}}{1 \text{ ounce}} \end{aligned}$$

The unit price is 6¢ per ounce.

7-3 Algebra: Solving Proportions (pp. 297–300)

Solve each proportion.

25. $\frac{x}{10} = \frac{3}{5}$

26. $\frac{4}{9} = \frac{24}{m}$

27. $\frac{2}{t} = \frac{8}{50}$

28. $\frac{15}{w} = \frac{35}{21}$

29. $\frac{12}{8} = \frac{a}{6}$

30. $\frac{7}{18} = \frac{d}{6}$

31. **MODELS** A Boeing 747 jet is 70.5 meters long and has a wingspan of 60 meters. A model of the 747 has a wingspan of 80 centimeters. What is its length?

Example 4 Solve $\frac{6}{9} = \frac{n}{12}$.

$$\frac{6}{9} = \frac{n}{12}$$

Write the proportion.

$$6 \times 12 = 9 \times n$$

Find the cross products.

$$72 = 9n$$

Multiply.

$$\frac{72}{9} = \frac{9n}{9}$$

Divide each side by 9.

$$8 = n$$

Simplify.

The solution is 8.

7-4 Geometry: Scale Drawings (pp. 304–308)

Suppose you are making a scale drawing. Find the length of each object on the drawing with the given scale.

32. a minivan 10 feet long;
2 inches = 1 foot
33. a rose garden 9 meters wide;
3 centimeters = 1 meter
34. a Ferris wheel with a diameter of 42 feet; 2 inches = 3 feet
35. a library 10.4 meters wide;
1 centimeter = 2 meters

Example 5 On a map, the distance between Chicago, Illinois, and Mexico City, Mexico, is 10.9 centimeters. If the scale is 1 centimeter = 250 kilometers, what is the actual distance?

$$\begin{array}{l} \text{Scale} \qquad \text{Chicago to} \\ \qquad \qquad \text{Mexico City} \\ \frac{1 \text{ cm}}{250 \text{ km}} = \frac{10.9 \text{ cm}}{n \text{ km}} \end{array}$$

Write a proportion.

$$1 \times n = 250 \times 10.9$$

Find the cross products.

$$n = 2,725$$

Simplify.

The actual distance is 2,725 kilometers.

Mixed Problem SolvingFor mixed problem-solving practice,
see page 602.**7-5 Fractions, Decimals, and Percents** (pp. 312–315)

Write each percent as a fraction in simplest form.

36. 27.5% 37. 5.4% 38. $45\frac{1}{4}\%$

Write each fraction as a percent. Round to the nearest hundredth if necessary.

39. $\frac{1}{8}$ 40. $\frac{5}{6}$ 41. $\frac{7}{40}$

Example 6 Write 82.5% as a fraction in simplest form.

$$82.5\% = \frac{82.5}{100}$$

Write a fraction with a denominator of 100.

$$= \frac{825}{1,000}$$

Multiply 82.5 and 100 by 10 to eliminate the decimal.

$$= \frac{33}{40}$$

Simplify.

7-6 Percents Greater Than 100% and Percents Less Than 1% (pp. 316–318)

Write each percent as a decimal and as a mixed number or fraction in simplest form.

42. 125% 43. 0.75%
44. 563% 45. 0.5%

Write each decimal as a percent.

46. 0.002 47. 4.75
48. 7.5 49. 0.0095

Example 7 Write 235% as a decimal and as a mixed number in simplest form.

$$235\% = \frac{235}{100}$$

Definition of percent

$$= 2.35 \text{ or } 2\frac{7}{20}$$

Example 8 Write 0.008 as a percent.

$$0.008 = \frac{0.008}{1}$$

Multiply by 100.

$$= 0.8\%$$

7-7 Percent of a Number (pp. 319–321)

Find each number. Round to the nearest tenth if necessary.

50. Find 78% of 50.
51. 45.5% of 75 is what number?
52. What is 225% of 60?
53. 0.75% of 80 is what number?

Example 9 Find 24% of 200.

$$24\% \text{ of } 200$$

$$= 24\% \times 200$$

Write a multiplication expression.

$$= 0.24 \times 200$$

Write 24% as a decimal.

$$= 48$$

Multiply.

So, 24% of 200 is 48.

7-8 The Percent Proportion (pp. 323–325)

Find each number. Round to the nearest tenth if necessary.

54. 6 is what percent of 120?
55. Find 0.8% of 35.
56. What percent of 375 is 40?
57. Find 310% of 42.

Example 10 What percent of 90 is 18?

$$\frac{a}{b} = \frac{p}{100}$$

Percent proportion

$$\frac{18}{90} = \frac{p}{100}$$

Replace a with 18 and b with 90.

$$18 \cdot 100 = 90 \cdot p$$

Find the cross products.

$$20 = p$$

So, 18 is 20% of 90.

Practice Test

Vocabulary and Concepts

1. Define *equivalent ratios*.
2. Write a proportion that can be used to find 76% of 512.

Skills and Applications

Write each ratio as a fraction in simplest form.

3. 45 to 18
4. 24:88
5. 15 minutes to 2 hours

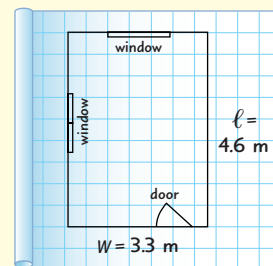
Find each unit rate. Round to the nearest hundredth if necessary.

6. 24 cards for \$4.80
7. 330 miles on 15 gallons of gasoline

Solve each proportion.

$$8. \frac{2}{3} = \frac{x}{42} \qquad 9. \frac{t}{21} = \frac{15}{14} \qquad 10. \frac{9}{m} = \frac{12}{36}$$

11. **BLUEPRINTS** The dimensions of a rectangular room in a new home are shown in the blueprint at the right. If the scale is 1 centimeter = 2.5 meters, find the length ℓ of the room on the scale drawing. Then find the scale factor.



12. **SWIMMING** Alyssa swims 3 laps in 12 minutes. At this same rate, how many laps will she swim in 30 minutes?

Write each percent as a fraction in simplest form.

13. $56\frac{1}{4}\%$
14. $82\frac{1}{4}\%$
15. $83\frac{1}{3}\%$

Write each fraction as a percent. Round to the nearest hundredth if necessary.

16. $\frac{5}{8}$
17. $\frac{7}{15}$
18. $\frac{33}{40}$

19. Write 0.45% as a decimal and as a fraction in simplest form.

20. Write 8.25 as a percent.

Find each number. Round to the nearest tenth if necessary.

21. What is 18% of 30?
22. What number is 162.2% of 50?
23. What percent of 64 is 12.8?
24. 458 is 105% of what number?

Standardized Test Practice

25. **MULTIPLE CHOICE** The purchase price of a bicycle is \$140. The state tax rate is $6\frac{1}{2}\%$ of the purchase price. What is the tax rate expressed as a decimal?

- (A) 6.5 (B) 0.65 (C) 0.065 (D) 0.0065



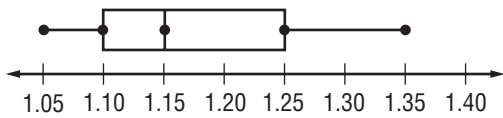
PART 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Andre charges \$4 per day for taking care of the neighbor's dog. Which expression represents the amount of money he earns in n days? (Lesson 1-4)

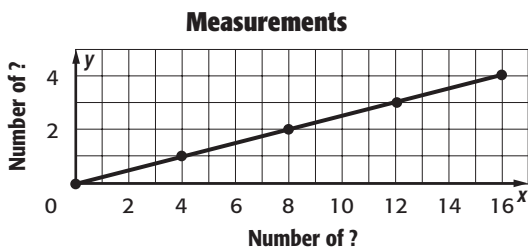
- (A) $4n$ (B) $4 + n$
 (C) $4 - n$ (D) n^4

2. The box-and-whisker plot shows the prices of single-scoop ice cream cones. What is the median price? (Lesson 2-6)



- (F) \$1.05 (G) \$1.15
 (H) \$1.25 (I) \$1.35

3. The graph shows the relationship between which two units of measure? (Lessons 4-6 and 6-7)



- (A) feet and inches
 (B) yards and feet
 (C) pounds and ounces
 (D) quarts and gallons

4. At Igoe's Campsite, 5 of the 15 sites were rented by campers. Which decimal equals the fraction of sites that were rented? (Lesson 5-4)

- (F) $0.\overline{33}$ (G) 0.30 (H) 3.30 (I) 33.30

5. A survey showed that the ratio of tourists at the Grand Canyon who had cameras to those who did not was 8 : 10. A similar survey was conducted at the Statue of Liberty. If the ratios are equivalent, what could be the ratio at the Statue of Liberty? (Lesson 7-1)

- (A) 32 : 30 (B) 30 : 24
 (C) 30 : 32 (D) 24 : 30

6. Tre can play 18 holes of golf in 180 minutes. What is his average rate in number of minutes per hole? (Lesson 7-2)

- (F) 1.8 (G) 3
 (H) 6 (I) 10

7. Which of the following is the value of p in $\frac{p}{7} = \frac{21}{49}$? (Lesson 7-3)

- (A) 3 (B) 6
 (C) 11 (D) 16

8. A scale model of a computer circuit board is 1 centimeter = 0.2 centimeter. The actual circuit board is 4 centimeters wide. What is the width of the model? (Lesson 7-4)

- (F) 0.2 cm (G) 2 cm
 (H) 20 cm (I) 200 cm

9. The table shows the percent of Internet users in each region of the United States who are 3–17 years old. What fraction of Internet users in the South region are 3–17 years old? (Lesson 7-5)

Internet Users 3–17 Years Old	
Region	Percent
Midwest	32
Northeast	36
South	28
West	29

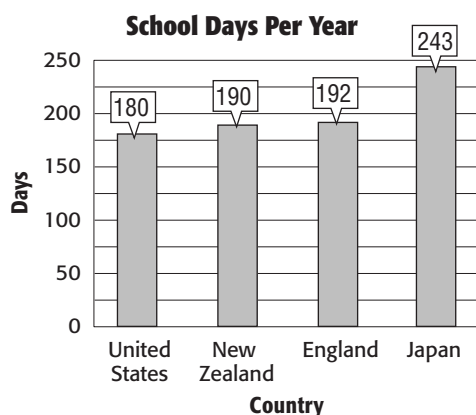
Source: U.S. Census Bureau

- (A) $\frac{13}{50}$ (B) $\frac{7}{25}$
 (C) $\frac{8}{25}$ (D) $\frac{36}{50}$

PART 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. The graph shows the number of school days per year in four countries. How many more days per year do Japanese students attend school than students in the United States? (Lesson 2-7)



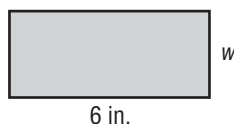
11. A train goes 65 miles per hour and travels 320 miles. How many hours will it take for the train to reach its destination? Round to the nearest tenth. (Lesson 3-7)

12. The table at the right represents a function. What is the missing x -coordinate? (Lesson 4-6)

x	y
3	9
4	12
7	21
?	27

13. Mellia has written $\frac{3}{5}$ of a research paper on fine art and French literature. What percent of the paper has she written? (Lesson 5-5)

14. The area of the rectangle is 18 square inches. What is the width w ?



15. Ron's camera has a shutter speed of $\frac{1}{1,000}$. What is this shutter speed expressed as a percent? (Lesson 7-6)

16. Jean practices her headstand in gymnastics class every day. She has been able to stay in the headstand pose without tipping for 45% of the last 60 classes. How many classes is this? (Lesson 7-7)
17. The table shows the nutrition amounts in one serving of pretzels and the percent of the recommended daily amounts.

Nutrition Facts of Pretzels		
Item	Amount	Percent*
fat	1 g	1.5%
sodium	486 mg	20%
carbohydrate	22 g	7.5%

Source: www.diet-data.com
*based on a 2,000-Calorie diet

How many daily milligrams of sodium are recommended? (Lesson 7-8)

PART 3 Extended Response

Record your answers on a sheet of paper. Show your work.

18. Jessie built a model of her town using items from her family's recycling bin.
- One centimeter in Jessie's model equals 10 kilometers in the real town. If Lansdowne Road is 12 centimeters long in the model, how long is the real Lansdowne Road? (Lesson 7-4)
 - Jessie has cut up milk cartons to construct $\frac{4}{5}$ of her model town. What percent is this? (Lesson 7-5)
 - Jessie used a total of 70 recycled items to build her model town. Explain how to find the number of items that were milk cartons. Then solve. (Lesson 7-8)

TEST-TAKING TIP

Question 18 When a question involves information from a previous part of a question, make sure to check that information before you move on.

