# UNIT 2 <br> <br> integers and Algebra 

 <br> <br> integers and Algebra}

## Gmapter 3

Algebra: Integers

## Chlater 4

Algebra: Linear Equations and Functions

Your study of math includes more than just whole numbers and decimals. In this unit, you will use negative numbers to describe many real-life situations and you will solve and graph equations that represent them.


CHAPTER
Algetra: lutegers


What does lightning have to do with math?
Have you heard of the phrase "opposites attract"? During thunderstorms, negatively-charged electrons in the clouds are attracted to positively-charged protons on the ground. This opposite attraction causes lightning. In mathematics, you can use opposites to help you add and subtract positive and negative integers.
You will solve problems about thunderstorms in Lesson 3-4.

## GETTING STARTED

## Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 3. Refer to the lesson or page number in parentheses for review.

## Vocabulary Review

State whether each sentence is true or false. If false, replace the underlined word to make a true sentence.

1. The mean of 1,3 , and 6 is 3 .
(Lesson 2-4)
2. The difference between the greatest number and the least number in a set of data is called the range. (Lesson 2-3)

## Prerequisite Skills

Replace each with < or > to make a true sentence. (Page 556)
3. $1,458 \bigcirc 1,548$
4. $36 \bigcirc 34$
5. $1.02 \bigcirc 1.20$
6. $76.7 \bigcirc 77.6$

Add.
7. $84+39$
8. $198+289$
9. $826+904$
10. $3,068+5,294$

## Multiply.

11. $2 \cdot 5 \cdot 3$
12. $18 \cdot 9$
13. $15 \cdot 6$
14. $10 \cdot 4 \cdot 7$

Divide.
15. $63 \div 9$
16. $96 \div 12$
17. $125 \div 5$
18. $187 \div 17$

Find the mean and range for each set of data. (Lessons 2-3 and 2-4)

egers Make this Foldable to help you organize information about integers. Begin with two sheets of $8 \frac{1}{2}^{\prime \prime} \times 11^{\prime \prime}$ paper.

## STEP 1 Fold and Cut One Sheet

Fold in half from top to bottom. Cut along fold from edges to margin.


## STEP2

Fold and Cut the Other Sheet
Fold in half from top to bottom. Cut along fold between margins.

## STEP 3 Fold

Insert first sheet through second sheet and align folds.

## STEP4 Label

Label each page with a lesson number and title.


## Votealiles

Chapter Notes Each time you find this logo throughout the chapter, use your Noteables ${ }^{\text {T0 }}$ : Interactive Study Notebook with Foldables ${ }^{\text {TM }}$ or your own notebook to take notes. Begin your chapter notes with this Foldable activity.

## Integers and Absolute Value

## WHEN

am I ever going to use this?

## What You'Il LEARN

Read and write integers, and find the absolute value of an integer.

## NEW Vocabulary

## integer

graph positive integer negative integer absolute value

## MATH Symbols

+3 positive three
-3 negative three
$|3|$ absolute value of three

Numbers like 9 and -2 are called integers. An integer is any number from the set $\{\ldots,-4,-3,-2,-1,0,1,2,3,4, \ldots\}$. Integers can be graphed on a number line. To graph a point on the number line, draw a point on the line at its location.


## EXAMPLES Write Integers for Real-Life Situations

WEATHER Write an integer for each situation.
(1) The average temperature in Tennessee for May was 5 degrees below normal.

Because it represents below normal, the integer is -5 .
(2) The average rainfall in Virginia for November was 5 inches above normal.
Because it represents above normal, the integer is +5 or 5 .

## Your Turn write an integer for each situation.

a. 6 degrees above normal
b. 2 inches below normal

## READING Math

Set Theory The number 5 is an element, or member, of the set of integers. The set $\{-5,5\}$ is a subset of the set of integers.

You can represent the integers from Examples 1 and 2 on a number line.

The numbers -5 and 5 are the same distance from 0 , but on opposite sides of 0 . So, -5 and 5 have the same absolute value.

## Notentinlesia

Key Concept: Absolute Value
Words The absolute value of an integer is the distance the number is from zero on a number line.
Examples $|6|=6 \quad|-6|=6$

## EXAMPLES Evaluate Expressions

## Evaluate each expression.

(3) $|-4|$

On the number line, the graph of -4 is 4 units from 0 .
So, $|-4|=4$.

(4) $|-5|-|2|$
$|-5|-|2|=5-2 \quad|-5|=5,|2|=2$
$=3$ Subtract.

## Skill and Concept Check

1. Writing Math Describe a situation in everyday life where negative numbers are used.
2. OPEN ENDED On a number line, graph two different points that have the same absolute value.
3. Which One Doesn't Belong? Identify the expression that does not have the same value as the other three. Explain your reasoning.

| $\|-3\|$ | -3 | $\|3\|$ |
| :--- | :--- | :--- |

## GUIDED PRECTICE

## Write an integer for each situation.

4. $6^{\circ} \mathrm{F}$ below 0
5. a loss of 11 yards
6. a deposit of $\$ 16$

Evaluate each expression.
7. $|7|$
8. $|-4|$
9. $|-7|-|1|$
10. STOCK MARKET The price of a company's stock fell 21 points in two days. Write an integer to represent the amount the stock price fell.

Write an integer for each situation.
11. a profit of $\$ 9$
13. no gain on first down
15. 2008 A.D.
17. a bank withdrawal of $\$ 50$
12. $53^{\circ} \mathrm{C}$ below 0
14. an elevator goes up 12 floors
16. 160 feet above sea level
18. 1000 в.с.

For Exercises See Examples

| $11-18,29$ | 1,2 |
| :---: | :---: |
| $19-22,27,28$ | 3 |
| $23-26$ | 4 |
| Extra Practice |  |
| See pages $569,598$. |  |

Evaluate each expression.
19. $|6|$
20. $|-12|$
21. $|-9|$
22. $|21|$
23. $|12|-|-8|$
24. $|-10|-5$
25. $|-9|+|5|$
26. $|26|+|-4|$
27. What is the absolute value of 0 ?
28. Find $|x|$ if $x=-6$.
29. STATIC ELECTRICITY Electrical charges are made up of positively-charged protons and negatively-charged electrons. Suppose you rub a balloon through your hair to make the balloon stick to a wall. There are 2 protons on the wall and 5 electrons on the balloon. Write an integer for each charge.

Graph each set of integers on a number line.
30. $\{0,1,-3\}$
31. $\{-4,5,4\}$
32. $\{-5,-1,10,-9\}$
33. $\{-2,-4,-6,-8\}$
34. WEATHER A meteorologist reports a $20^{\circ}$ change in the temperature from yesterday to today. Describe what this could mean.

CRITICAL THINKING Determine whether each statement is true or false.
If false, give a counterexample.
35. Every integer has an absolute value.
36. The absolute value of every integer is positive.

38. SHORT RESPONSE Write an integer for $23^{\circ} \mathrm{F}$ below 0 .

STATISTICS For Exercises 39 and 40, use the following information.
The mean income for a group of accountants is $\$ 36,266.67$. Their incomes are $\$ 27,500, \$ 36,100, \$ 29,800, \$ 33,400, \$ 31,300$, and $\$ 59,500$.
39. In what way is the mean misleading? (Lesson 2-8)
40. Draw a bar graph of the data. (Lesson 2-7)

GETTING READY FOR THE NEXT LESSON
PREREQUISITE SKILL Replace each with $<$ or $>$ to make a true sentence. (Page 556)
41. $16 \bigcirc 6$
42. $2.3 \bigcirc 3.2$
43. $101 \bigcirc 111$
44. $87.3 \bigcirc 83.7$
45. $1,051 \bigcirc 1,015$

## Comparing and Ordering Integers

## What You'll LEARN

Compare and order integers.

## REVIEW Vocabulary

median: the middle number in an ordered data set (Lesson 2-4)

## MATH Symbols

$<$ is less than
$>$ is greater than

## Whte am I ever going to use this?

## WEATHER The Wind Chill

Temperature Index table shows how cold air feels on human skin.

1. What is the wind chill if there is a wind at 20 miles per hour and the temperature is $5^{\circ}$ ?
2. Which is colder, a temperature of $15^{\circ}$ with a 20 mile-per-hour wind or a temperature of $10^{\circ}$

| WIND CHILL |  |  |  |  |  |
| :---: | ---: | ---: | :---: | :---: | :---: |
| Wind <br> $(\mathrm{mph})$ | Temperature ( ${ }^{\circ} \mathrm{F}$ ) |  |  |  |  |
|  | 15 | 10 | 5 | 0 | -5 |
| 5 | 7 | 1 | -5 | -11 | -16 |
| 10 | 3 | -4 | -10 | -16 | -22 |
| 15 | 0 | -7 | -13 | -19 | -26 |
| 20 | -2 | -9 | -15 | -22 | -29 | with a 10 mile-per-hour wind?

3. Graph both wind chills found in Exercise 2 on a number line.

When two numbers are graphed on a number line, the number to the left is always less than the number to the right. The number to the right is always greater than the number to the left.

Notealiflesil Key Concept: Compare Integers
Model


Words $\quad-4$ is less than $-2 . \quad-2$ is greater than -4 .
Symbols


The symbol points to the lesser number.

## EXAMPLE Compare Integers

(1) Replace the with $<$ or $>$ to make $-5-3$ a true sentence.

Graph each integer on a number line.


Since -5 is to the left of $-3,-5<-3$.

- Your Turn Replace each with $<$ or $>$ to make a true sentence.
a. $-8 \bigcirc-4$
b. $5-1$
c. $-10 \bullet-13$


## READING in the Content Area <br> For strategies in reading this lesson, visit msmath2.net/reading.

Integers are used to compare numbers in many real-life situations.

## EXAMPLE Order Integers

## Test-Taking Tip

## Eliminating Answer Choices

If you are unsure of the correct answer, eliminate the choices you know are incorrect. Then consider the remaining choices. You can eliminate choice C since the list begins with a positive number.

MULTIPLE-CHOICE TEST ITEM The lowest temperatures in Alaska, Florida, Hawaii, and Montana are listed in the table. Order the temperatures from least to greatest.

$$
\begin{aligned}
& \text { (A) }-80,-70,12,-2 \\
& \text { (B) }-80,-70,-2,12 \\
& \text { (C) } 12,-2,-70,-80 \\
& \text { (D) }-2,12,-70,-80
\end{aligned}
$$

| State | Record Low <br> Temperature ( ${ }^{\circ} \mathrm{F}$ ) |
| :---: | :---: |
| Alaska | $\mathbf{- 8 0}$ |
| Florida | $\mathbf{- 2}$ |
| Hawaii | $\mathbf{1 2}$ |
| Montana | $\mathbf{- 7 0}$ |
|  |  |

Source: The World Almanac and Book of Facts

Read the Test Item To order the integers, graph them on a number line.

## Solve the Test Item



Order the integers from least to greatest by reading from left to right: $-80,-70,-2,12$. So, the answer is $B$.

## Still and Concept Check

1. Draw a number line to show that -5 is less than -1 .
2. OPEN ENDED Write an integer that is less than -9 . Explain.
3. NUMBER SENSE Complete the sentence: -7 is greater than -12 because
-7 lies to the $\qquad$ of -12 on a number line.
4. Name the greatest negative integer.

## CuIDTO PRACTICE

Replace each with $<$ or $>$ to make a true sentence.
5. $-5-6$
6. -2 - 8
7. $0 \bigcirc-10$
8. Order 51, $-63,49,-24,-38$, and 38 from least to greatest.

MONEY For Exercises 9 and 10, use the information below and at the right.
Marva is saving money for a new bike and has already saved $\$ 21$. She begins a log to keep track of her money.
9. Write each entry as an integer.
10. Order the integers from least to greatest.


## Practice and Applications

Replace each with $<$ or $>$ to make a true sentence.
11. $-17-20$
12. $-21--12$
13. $3-10$
14. $-5 \bigcirc 17$
15. $4 \bigcirc-4$
16. $-25-20$
17. $-52--72$
18. $100-10$
19. $|-8| \odot 0$
20. $-13 \bullet|-14|$
21. $|36|--37$
22. $|-29|-|92|$

Determine whether each sentence is true or false. If false, change one number to make the sentence true.
23. $-8>5$
24. $-7<0$
25. $|-9|=9$
26. $|5|<-6$
27. $10>|-8|$
28. $7>|-7|$

WEATHER For Exercises 29 and 30, use the information in the table. It shows the record low temperatures in Indianapolis, Indiana, for March 1-7 of a recent year.
29. Arrange the dates from the coldest temperature to the warmest.
30. Find the median temperature.
31. Order $-7,5,-6,-4,1$, and 3 from least to greatest.
32. Order $|51|,-53,|-52|, 55,-56$, and -57 from greatest to least.

| Day | Temperature ${ }^{\circ} \mathrm{P}$ ? |
| :---: | :---: |
| Mar. 1 | $\mathbf{- 4}$ |
| Mar. $\mathbf{2}$ | $\mathbf{- 7}$ |
| Mar. $\mathbf{3}$ | $\mathbf{1}$ |
| Mar. $\mathbf{4}$ | $\mathbf{3}$ |
| Mar. $\mathbf{5}$ | $\mathbf{- 1}$ |
| Mar. $\mathbf{6}$ | $\mathbf{- 6}$ |
| Mar. $\mathbf{7}$ | $\mathbf{6}$ |

33. CRITICAL THINKING If 0 is the greatest integer in a set of five

Source: www.weather.com integers, what can you conclude about the other four integers?

## Spiral Review with Standardized Test Practice

34. MULTIPLE CHOICE The table shows the inventions of several toys. Order the inventions from earliest to most recent.
(A) chess, yo-yo, teddy bear, checkers
(B) checkers, yo-yo, chess, teddy bear
(C) yo-yo, teddy bear, checkers, chess

| Toy | Year |
| :--- | ---: |
| Yo-Yo | 1000 B.c. |
| Teddy Bear | 1902 A.D. |
| Chess | 600 A.d. |
| Checkers | 2000 B.c. |

(D) chess, teddy bear, yo-yo, checkers
35. GRID IN Which is greater, -12 or 7 ?

Write an integer for each situation. (Lesson 3-1)
36. $9^{\circ} \mathrm{C}$ below 0
37. a gain of 20 feet
38. PROFITS The daily profits of T-shirts sold last week were $\$ 55, \$ 35, \$ 25$, $\$ 30$, and $\$ 55$. Which average might be misleading: the mode, the median, or the mean? Explain. (Lesson 2-8)

## GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Graph the solution of each equation on a number line. (Lesson 1-5)
39. $x+3=5$
40. $x-4=8$
41. $3 x=9$
42. $5 x=30$

## Geometry: The Coordinate Plane

## What You'll LEARN

Graph points on a coordinate plane.

## NEW Vocabulary

coordinate plane coordinate grid $x$-axis
$y$-axis
origin
ordered pair $x$-coordinate $y$-coordinate quadrant

## When am I ever going to use this?

MAPS A map of Terrell's neighborhood is shown.

1. Suppose Terrell starts at the corner of Russel and Main and walks 1 block north and 2 blocks east. Name the intersection of his location.
2. Using the words north, south, west, and east, write directions to go from the corner of School and Highland to the corner of Main and Oak.

A coordinate plane is used to locate points. It is a plane in which a horizontal number line and a vertical number line intersect at their zero points. A coordinate plane is also called a coordinate grid.


An ordered pair is a pair of numbers such as $(5,-2)$ used to locate a point in the coordinate plane.
The $\boldsymbol{x}$-coordinate corresponds
to a number on the $x$-axis.

## EXAMPLE Name an Ordered Pair

(1) Name the ordered pair for point $P$.

- Start at the origin.
- Move left to find the $x$-coordinate of point $P$, which is -4 .
- Move up to find the $y$-coordinate, which is 2 .

So, the ordered pair for point $P$ is $(-4,2)$.

## READING Math

Scale When no numbers are shown on the $x$ - or $y$-axis, you can assume that each grid is 1 unit long on each side.

EXAMPLE Graph an Ordered Pair

1) Graph and label the point $Q(2,-5)$.

- Draw a coordinate plane.
- Move 2 units to the right. Then move 5 units down.
- Draw a dot and label it $Q(2,-5)$.



## - Your Turn Graph each point.

a. $A(6,0)$
b. $B(-5,-3)$

The coordinate plane is separated into four sections called quadrants.


Axes is the plural form of axis. The axes are not located in any of the quadrants.

## REALE CAREERS

## How Does a

Cartographer Use Math?
Cartographers use coordinates to prepare geographic, political, and cultural maps of large areas.

Research
For more information about a career as a cartographer, visit: msmath2.net/careers


## EXAMPLES Identify Quadrants

(3) GEOGRAPHY The world map can be divided into a coordinate grid where ( $x, y$ ) represents (degrees longitude, degrees latitude). In which quadrant is the United States located?


The United States is located in the upper left quadrant, quadrant II.
D Name a country from the map that is located in quadrant III. Quadrant III is the bottom-left quadrant. So, Chile is in quadrant III.

## Skill and Concept Check

1. 



Explain why point $A(1,-2)$ is different from point $B(-2,1)$.
2. OPEN ENDED Name and graph a point in quadrant IV.

## CUIDRD PRACTICE

Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.
3. $P$
4. $Q$
5. $R$

On graph paper, draw a coordinate plane. Then graph and label each point.
6. $S(2,3)$
7. $T(-4,6)$
8. $U(-5,0)$


## Practice and Applications

Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.
9. $A$
10. $B$
11. $C$
12. $D$
13. $E$
14. $F$
15. $G$
16. $H$
17. I
18. $J$


## HOMEWORK HELP

For Examples See Examples 9-18 1,3 19-31 2 35-42 3,4 Extra Practice See pages 570, 598.
19. Write the ordered pair for the point that lies on the $y$-axis and is 32 units down from the origin.

On graph paper, draw a coordinate plane. Then graph and label each point.
20. $M(5,6)$
21. $N(-2,10)$
22. $P(7,-8)$
23. $Q(3,0)$
24. $R(-1,-7)$
25. $S(0,6)$
26. $T(-3,7)$
27. $U(5,-2)$
28. $V(8,1)$
29. $W(-5,-7)$
30. $X(1.5,-3)$
31. $Y(-6.5,6.5)$

Determine whether each statement is sometimes, always, or never true.
Explain or give a counterexample to support your answer.
32. Both $x$ - and $y$-coordinates of a point in quadrant $I$ are negative.
33. The $x$-coordinate of a point that lies on the $x$-axis is negative.
34. The $y$-coordinate of a point in quadrant IV is negative.

GEOGRAPHY For Exercises 35 and 36, use the map in Example 3.
35. In what country is the point ( $105^{\circ}$ longitude, $30^{\circ}$ latitude) located?
36. Find an ordered pair that can represent the location of California.

For Exercises 37-41, use the map of the Brookfield Zoo.
37. What exhibit is located at $(4,-2)$ ?
38. In which quadrant is the Dragonfly Marsh exhibit located?
39. Find the ordered pair that represents the location of Baboon Island.
40. What is located at the origin?

Brookfield Zoo, Brookfield, IL


Source: www.brookfieldzoo.org
41. Describe how you would walk from the entrance of the Pachyderm

House at $(-2,2)$ to the entrance of The Swamp at $(-1,-2)$.
42. GEOMETRY Graph the points $A(-3,2), B(2,2), C(2,-4)$, and $D(-3,-4)$ on the same coordinate plane. Connect the points from $A$ to $B, B$ to $C$, $C$ to $D$, and $D$ to $A$. Name the figure.
43. CRITICAL THINKING Find the possible locations for any ordered pair whose $x$ - and $y$-coordinates are always the same integer. Explain.

## Spiral Review with Standardized Test Practice

For Exercises 44-46, use the coordinate plane at the right.
44. MULTIPLE CHOICE Which building has the coordinates of $(-3,-1)$ ?
(A) museum (B) park (C) library (D) aquarium
45. MULTIPLE CHOICE What are the coordinates of the point that shows the location of the hotel?
(F) $(-1,-3)$
(G) $(1,-3)$
(H) $(-1,3)$
(I) $(1,3)$
46. SHORT RESPONSE In which quadrant is the grocery store located?


Replace each with $<$ or $>$ to make a true sentence. (Lesson 3-2)
47. $14 \bigcirc|-15|$
48. $-8-3$
49. $26-30$
50. $-40 \bullet|40|$
51. Find the absolute value of -101 . (Lesson $3-1$ )
52. SPORTS A triathlon competition consists of swimming 3 miles, running 10 miles, and bicycling 35 miles. How many miles does an athlete travel during the competition? (Lesson 1-1)

## GETTING READY FOR THE NEXT LESSON

BASIC SKILL Add.
53. $138+246$
54. $814+512$
55. $2,653+4,817$
56. $6,003+5,734$

## m <br> 3) <br> Mid-Chapter Practice Test

## Vocabulary and Concepts

1. Define absolute value. (Lesson 3-1)
2. Write the ordered pair which identifies a point 4 units to the left of the $y$-axis and three units above the $x$-axis. (Lesson 3 -3)
3. Draw a coordinate plane, and label the quadrants. (Lesson $3-3$ )

## Skills and Applications

Write an integer for each situation. (Lesson 3-1)
4. 45 feet below sea level
5. a deposit of $\$ 100$
6. a gain of 8 yards
7. lost a $\$ 5$ bill

Replace each with $<$ or $>$ to make a true sentence. (Lesson 3-2)
8. $-12-9$
9. $-4 \bigcirc 4$
10. $|-14| \odot|3|$
11. FOOTBALL The Tigers have recorded the following yardage on the past six plays: $9,-2,5,0,12$, and -7 . Order the integers from least to greatest. (Lesson 3-2)

On graph paper, draw a coordinate plane. Then graph and label each point. (Lesson 3-3)
12. $D(4,-3)$
13. $E(1,3)$
14. $F(0,-5)$

## Standardized Test Practice

15. MULTIPLE CHOICE Which of the following points represents a number and its absolute value?
(Lesson 3-1)

(A) $B$ and $E$
(B) $C$ and $F$
(c) $B$ and $D$
(D) $A$ and $E$
16. SHORT RESPONSE The table shows the number of inches of monthly precipitation above or below normal for a midwestern city in a recent year. Find the median monthly precipitation above or below normal. (Lesson 3-2)

| Monthly Precipitation <br> Above or |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| J | F | M | A | M | J |
| 4 | -1 | 6 | -2 | -3 | 1 |
| J | A | S | 0 | N | D |
| -2 | -1 | 2 | -3 | 1 | -3 |

## Adding Integers

## What You'll LEARN

Use counters to model the addition of integers.

## Waterials

- counters
- integer mat

You can use positive and negative counters to model the addition of integers. The counter $\oplus$ represents 1 , and the counter $\Theta$ represents -1 . Remember that addition means combining two sets.


## ACTIVITY

## Work with a partner.

Use counters to find $-2+(-4)$.
 counters and a set of 4 negative counters.


Find the total number of counters.


So, $-2+(-4)=-6$.

## Your Turn Use counters to find each sum.

a. $5+6$
b. $-3+(-5)$
c. $-5+(-4)$
d. $7+3$
e. $-2+(-5)$
f. $-8+(-6)$

The following two properties are important when modeling operations with integers.

- When one positive counter is paired with one negative counter, the result is called a zero pair. The value of a zero pair is 0 .
- You can add or remove zero pairs from a mat because adding or removing zero does not change the value of the counters on the mat.
You will use zero pairs in Activity 2 and Activity 3.



## Writing Math

1. Write two addition sentences where the sum is positive. In each sentence, one addend should be positive and the other negative.
2. Write two addition sentences where the sum is negative. In each sentence, one addend should be positive and the other negative.
3. MAKE A CONJECTURE Write a rule that will help you determine the sign when finding the sum of integers.

## 3-4 <br> Adding Integers

## What You'll LEARN

Add integers.

## NEW Vocabulary

opposites
additive inverse

## Link to READING

## Everyday Meaning of

 Opposite: something that is across from or is facing the other way, as in running the opposite way
## When am I ever going to use this?

EARTH SCIENCE Thunderstorms are made of both positive and negative electrical charges. The negative charges (electrons) are at the bottom of a thundercloud, and positive charges (protons) are at the top.

1. What is the charge at the top of a cloud where there are more protons than electrons?

2. What is the charge at the bottom of a cloud where there are more electrons than protons?

Combining positive and negative electrical charges in a thunderstorm is similar to adding integers.

## EXAMPLE Add Integers with the Same Sign

(1) Find $-3+(-2)$.

Use a number line.

- Start at 0 .
- Move 3 units left to show -3.

- From there, move 2 units left to show -2 .

So, $-3+(-2)=-5$.

## Voteatiblest

Key Concept: Add Integers with the Same Sign
Words The sum of two positive integers is positive.
The sum of two negative integers is negative.
Examples $7+4=11 \quad-7+(-4)=-11$

Technology To enter a negative integer on a calculator, use the $(-)$ key. For example, to enter -5 , press $(-)$.

## EXAMPLE Add Integers with the Same Sign

(2) Find $-26+(-17)$.
$-26+(-17)=-43$ The sum of two negative integers is negative.

## Your Turn add.

a. $-14+(-16)$
b. $23+38$
c. $-35+(-49)$

The integers 43 and -43 are called opposites of each other because they are the same distance from 0 , but on opposite sides of 0 . Two integers that are opposites are also called additive inverses.

## Notenifilesit

Key Concept: Additive Inverse Property
Words The sum of any number and its additive inverse is 0 .
Example $5+(-5)=0$

## EXAMPLES Add Integers with Different Signs

(3) Find $5+(-3)$.

Use counters.
Remove all zero pairs.


So, $5+(-3)=2$.

Find $-3+2$
Use a number line.

- Start at 0 .
- Move 3 units left.
- Then move 2 units right.


So, $-3+2=-1$.

Look Back You can review absolute value in Lesson 3-1.

## Noteainlesil

Key Concept: Add Integers with Different Signs
Words To add integers with different signs, subtract their absolute values. The sum is:

- positive if the positive integer has the greater absolute value.
- negative if the negative integer has the greater absolute value.

Examples $9+(-4)=5 \quad-9+4=-5$

## EXAMPLES

Add Integers with Different Signs
(5) Find $7+(-1)$.
$7+(-1)=6 \quad$ Subtract. Keep the sign of the integer with the greater absolute value.
B Find $-8+4$.
$-8+4=-4 \quad$ Subtract. Keep the sign of the integer with the greater absolute value.

## - Your Turn add.

d. $10+(-12)$
e. $-16+9$
f. $-13+18$

## EXAMPLE Simplify an Expression with Integers

(1) ALGEBRA Simplify $12+x+(-20)$.

$$
\begin{aligned}
12+x+(-20) & =12+(-20)+x & & \text { Commutative Property of Addition } \\
& =-8+x & & \text { Add. }
\end{aligned}
$$

- Your Turn simplify each expression.


## REAL-LIFE MATH

g. $1+y+(-5)$
h. $z+(-2)+9$

OCEANOGRAPHY The
Great Barrier Reef off the coast of Australia is home to 1,500 species of fish and 215 species of birds.
Source: www.ozramp.net.au

## EXAMPLE Use Integers to Solve a Problem

- OCEANOGRAPHY Anna was scuba diving near the Great Barrier Reef 16 meters below the surface of the water. She saw a dolphin swim by 7 meters above her. What was the depth of the dolphin? Anna is 16 meters underwater, and the dolphin is 7 meters above her. So, the depth of the dolphin can be represented by the expression $-16+7$, or -9 .
The dolphin is 9 meters below the surface of the water.


## Skill and Concept Check

1. Draw a model to show $2+(-7)$.
2. OPEN ENDED Give an example of integers that are additive inverses.
3. FIND THE ERROR Brooke and Javier are finding $-12+13$. Who is correct? Explain.

> Brooke
> $-12+13=1$
Javier $-12+13=-1$
4. NUMBER SENSE Tell whether each sum is positive, negative, or zero without adding.
a. $-6+(-7)$
b. $-8+10$
c. $-14+14$

## CuIDED PRACTICE

Add.
5. $-6+(-8)$
6. $-3+10$
7. $7+(-11)$
8. $9+(-9)$
9. MONEY You pay your brother $\$ 42$ that you owe him. The same week you earn $\$ 35$ dog-sitting for the neighbors. Do you have more or less money than at the beginning of the week?
10. ALGEBRA Simplify $12+y+(-8)$.

Add.
11. $-8+8$
12. $-9+11$
13. $13+(-19)$
14. $6+10$
15. $-10+(-15)$
16. $-12+10$
17. $-30+16$
18. $18+(-5)$
19. $21+(-21)$
20. $18+(-20)$
21. $-22+(-16)$
22. $-24+19$
23. $-11+13+6$
24. $-16+(-21)+15$
25. $12+(-17)+(-25)$
26. $20+(-30)+(-40)$

| For Exercises | See Examples |
| :---: | :---: |
| $11-42$ | $1-6$ |
| $46-57$ | 7 |
| $43-45,58$ | 8 |

Write an addition expression to describe each situation. Then find each sum.
27. WEATHER The temperature outside is $-3^{\circ} \mathrm{F}$. The temperature drops $6^{\circ}$.
28. SUBMARINE A submarine dives 106 feet below the water. Then, it rises 63 feet.
29. SKATEBOARDING Hakeem starts at the bottom of a half pipe 6 feet below street level. He rises 14 feet at the top of his kickturn.
30. MONEY MATTERS Stephanie has $\$ 43$ in the bank. She withdraws $\$ 35$.

ALGEBRA Evaluate each expression if $x=-10, y=7$, and $z=-8$.
31. $x+14$
32. $6+y$
33. $z+(-5)$
34. $-17+y$
35. $20+z$
36. $-10+x$
37. $z+8$
38. $15+x$
39. $x+y$
40. $y+z$
41. $x+z$
42. $x+y+z$

GOLF For Exercises 43-45, use the information below.
Scores over par in a golf tournament are recorded as positive integers. Scores under par are recorded as negative integers. Even par is recorded as 0 . The person with the lowest total score wins. The table shows the top two finishers in the 2004 LPGA Championship.
43. Find Annika Sorenstam's final score.

|  | Round 1 | Round 2 | Round 3 | Round 4 |
| :--- | :---: | :---: | :---: | :---: |
| Annika Sorenstam | -3 | -4 | -7 | +1 |
| Shi Hyun Ahn | -2 | -1 | -2 | -5 |

Source: www.Ipga.com

Data Update What were the four-round scores of the latest winners of the LPGA Championship? Visit msmath2.net/data_update to learn more.

ALGEBRA Simplify.
46. $x+(-5)+1$
47. $4+y+(-2)$
48. $-9+m+(-6)$
49. $8+(-8)+n$
50. $-1+a+7$
51. $f+(-19)+11$

Explain how the Commutative and Associative Properties of Addition can help you find each sum mentally. Then find each sum.
52. $7+(-2)+(-7)$
53. $-6+9+(-4)$
54. $-5+(-6)+(-3)$
55. $8+10+(-8)$
56. $-5+(-7)+(-10)$
57. $8+(-9)+9$
58. STOCK MARKET The members of the Investment Club purchased a stock for $\$ 50$. The next day the price of the stock dropped $\$ 18$. On the second and third days, the price dropped another $\$ 16$ and then rose $\$ 21$. How much was the stock worth at the end of the third day?
59. WRITE A PROBLEM Write about a real-life problem using the addition sentence $-8+11=t$. Then solve the equation and explain what the solution represents.

CRITICAL THINKING For Exercises 60 and 61, use a number line to find each sum. Does the order of the addends make a difference? Explain.
60. $3+(-8)$ and $-8+3$
61. $[7+(-3)]+(-6)$ and $7+[-3+(-6)]$

## Spiral Review with Standardized Test Practice

62. MULTIPLE CHOICE In a game with a standard deck of cards and the scoring system at the right, three cards are dealt and added together to get a final score. Dylan is dealt the 4 of hearts, the king of spades, and the 3 of diamonds. What is his final score?
$\begin{array}{lllll}\text { (A) }-11 & \text { (B) }-3 & \text { (C) } 3 & \text { (D) } 9\end{array}$
63. SHORT RESPONSE Jeremy owes his sister $\$ 5$. Then he borrows $\$ 6$ more from her. Write the total amount he owes as an integer.

Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies. (Lesson $3-3$ )
64. J
65. K
66. L
67. $M$
68. Order $6,-3,0,4,-8,1$, and -4 from least to greatest.
(Lesson 3-2)

69. STATISTICS Construct a line plot for the following test scores: $81,83,75$, $81,82,81,75,82,82,86,83,81$, and 79 . (Lesson 2-3)

GETTING READY FOR THE NEXT LESSON
PREREQUISITE SKILL Find the range for each set of data. (Lesson 2-3)
70. $13,7,6,22,21$
71. $54,32,43,49,30$
72. $62,59,85,74,82$

## Use a Flowchart

## Taking Good Notes

Have you ever tried to solve a math problem and then realized you left out an important step? Try using a flowchart when you take notes to map out the steps you should follow.

A flowchart is like a map that tells you how to get from the beginning of a problem to the end.

| A diamond contains a question. You need to stop and make a decision. |  |
| :--- | :--- |
|  | A rectangle tells you what to do. |
|  | An oval indicates the beginning or end. |

Here's a flowchart for adding two integers. Just follow the arrows.


## SKILL PRACTICE

## Make a flowchart for each kind of problem.

1. rounding a decimal to a given place (See page 557.)
2. evaluating an expression using order of operations (See Lesson 1-3.)

## 3=5a

## dHANDS-ON LAB

## What You'll LEARN

Use counters to model the subtraction of integers.

## Subtracting Integers

You can also use counters to model subtraction of integers. Remember one meaning of subtraction is to take away.

## ACTIVITIES Work with a partner.

Use counters to find each difference.
6-4


So, $6-4=2$.
$3-(-2)$


Now you can remove 2 negative counters. Find the remaining number of counters.

## READING Math

Minuends, Subtrahends, and Differences In the subtraction sentence $-4-3=-7,-4$ is the minuend, 3 is the subtrahend, and -7 is the difference.


## Writing Math

Work with a partner.

1. Write two subtraction sentences where the difference is positive. Make sure you use a combination of positive and negative integers.
2. Write two subtraction sentences where the difference is negative. Make sure you use a combination of positive and negative integers.
3. MAKE A CONJECTURE Write a rule that will help you determine the sign of the difference of two integers.

## 3-5 Subtracting Integers

## What You'll LEARN

Subtract integers.

REVIEW Vocabulary
range: the difference between the greatest number and the least number in a set of data (Lesson 2-3)

## GHMOSON

## Work with a partner.

## Materials

- graph paper

The subtraction problems below are modeled on number lines.


$-1-4=-5$

1. Write a related addition sentence for each subtraction sentence.

Use a number line to find each difference. Write an equivalent addition sentence for each.
2. $1-5$
3. $-2-1$
4. $-3-4$
5. $0-5$
6. Compare and contrast subtraction sentences with their related addition sentences.

When you subtract 5, as shown in the Mini Lab, the result is the same as adding -5 . When you subtract 4 , the result is the same as adding -4 .


Votealifesi
Words To subtract an integer, add its opposite.
Example $4-9=4+(-9)=-5$

## EXAMPLES Subtract Positive Integers

## Subtract.

(1) $8-13$

$$
\begin{aligned}
8-13 & =8+(-13) & & \text { To subtract } 13, \text { add }-13 . \\
& =-5 & & \text { Simplify. }
\end{aligned}
$$

(2) $-10-7$

$$
\begin{aligned}
-10-7 & =-10+(-7) & & \text { To subtract 7, add }-7 . \\
& =-17 & & \text { Simplify. }
\end{aligned}
$$

## EXAMPLES Subtract Negative Integers

(3) Find 1 - ( -2 ).
$1-(-2)=1+2 \quad$ To subtract -2 , add 2 .

$$
=3 \quad \text { Simplify }
$$

(1) Find -10 - (-7).

$$
\begin{aligned}
-10-(-7) & =-10+7 & & \text { To subtract }-7 \text {, add } 7 . \\
& =-3 & & \text { Simplify. }
\end{aligned}
$$

Technology Use the - key to subtract a number. Use the (-) key to enter a negative number.

## - Your Turn subtract.

a. $4-(-12)$
b. $-15-(-5)$
c. $18-(-6)$

## EXAMPLE Evaluate an Expression

(5) ALGEBRA Evaluate $x-y$ if $x=-6$ and $y=5$.
$x-y=-6-5 \quad$ Replace $x$ with -6 and $y$ with 5 .
$=-6+(-5) \quad$ To subtract 5 , add -5 .
$=-11 \quad$ Simplify.

- Your Turn Evaluate each expression if $a=5, b=-8$, and $c=-9$.
d. $b-10$
e. $a-b$
f. $c-a$


## EXAMPLE Use Integers to Solve a Problem

EARTH SCIENCE
The legend on the sea-surface temperature map shows the minimum temperature at $-2^{\circ} \mathrm{C}$ and the maximum temperature at $31^{\circ} \mathrm{C}$. What is the range of temperatures on the map?


To find the range, or difference in temperatures, subtract the lowest temperature from the highest temperature.

$$
\begin{aligned}
31-(-2) & =31+2 & & \text { To subtract }-2, \text { add } 2 . \\
& =33 & & \text { Simplify. }
\end{aligned}
$$

So, the range of temperatures is $33^{\circ} \mathrm{C}$.

## Skill and Concept Check

1. Writing Math Explain how additive inverses are used in subtraction.
2. OPEN ENDED Write a subtraction sentence using integers. Then, write the equivalent addition sentence and find the sum.
3. FIND THE ERROR Bradley and Mitsu are finding $-16-(-19)$. Who is correct? Explain.

$$
\begin{aligned}
& \text { Bradley } \\
&-16-(-19)=-16+(-19) \\
&=-35
\end{aligned}
$$

$$
\begin{gathered}
\text { Mitsu } \\
-16-(-19)=-16+(19) \\
=3
\end{gathered}
$$

## GUTOTD PRECTICE

## Subtract.

4. $-4-8$
5. $14-17$
6. $14-(-10)$
7. $-3-(-1)$

ALGEBRA Evaluate each expression if $p=8, q=-14$, and $r=-6$.
8. $p-q$
9. $q-r$
10. $r-p$
11. METEOROLOGY The highest temperature ever recorded on Earth was $136^{\circ} \mathrm{F}$ in Libya. The lowest temperature was $-129^{\circ} \mathrm{F}$ in Antarctica. What is the range of the highest and lowest temperatures on Earth?

## Practice and Applications

## Subtract.

12. $-9-5$
13. $0-10$
14. $-8-9$
15. $17-13$
16. $27-(-8)$
17. $-25-(-5)$
18. $12-26$
19. $4-(-19)$
20. $-11-42$
21. $15-(-14)$
22. $-27-(-19)$
23. $-18-(-20)$
24. $31-48$
25. $-33-(-27)$
26. $52-(-52)$
27. $-44-(-41)$
28. $-2-9+7$
29. $6+(-1)-4$

## HOMEWORK HELP

30. What is -3 minus 4 ?
31. Find $-23-(-19)$.

ALGEBRA Evaluate each expression if $f=-6, g=7$, and $h=9$.
32. $5-f$
33. $h-(-9)$
34. $f-g$
35. $g-7$
36. $h-f$
37. $f-6$
38. $g-h$
39. $4-(-g)$
40. $-h-10$
41. $-f-h$
42. $f-g-h$
43. $h-g-f$
44. ALGEBRA Find $|a-b|$ when $a=-7$ and $b=11$.
45. GEOGRAPHY The Dead Sea's deepest part is 799 meters below sea level.

A plateau to the east of the Dead Sea rises to about 1,340 meters above sea level. What is the difference between the deepest part of the Dead Sea and the top of the plateau?

HISTORY For Exercises 46-48, use the timeline that shows the lives of three rulers of Rome.

46. How old was Augustus when he died?
47. Who lived the longest? How old was he when he died?
48. How many years were there between when Julius Caesar was born and when Tiberius died?

Determine whether each statement is sometimes, always, or never true.
Give an example or counterexample for each answer.
49. negative - positive $=$ negative
50. negative - negative $=$ positive
51. positive - positive $=$ positive
52. positive - negative $=$ negative
53. CRITICAL THINKING

True or False? When $n$ is a negative integer, $n-n=0$.

## Spiral Review with Standardized Test Practice

54. MULTIPLE CHOICE Find the correct subtraction sentence shown in the model.

(A) $2-(-6)=-4$
(B) $-6-2=-4$
(C) $-2-4=-6$
(D) $2-6=-4$
55. SHORT RESPONSE The temperatures on the moon vary from $-173^{\circ} \mathrm{C}$ to $127^{\circ} \mathrm{C}$. Find the range of temperatures.

Add. (Lesson 3-4)
56. $10+(-3)$
57. $-2+(-9)$
58. $-7+(-6)$
59. $-18+4$
60. In which quadrant do ordered pairs with a positive $x$-coordinate and a negative $y$-coordinate lie? (Lesson 3-3)

## GETTING READY FOR THE NEXT LESSON

BASIC SKILL Multiply.
61. $14 \cdot 5$
62. $9 \cdot 16$
63. $6 \cdot 8 \cdot 4$
64. $11 \cdot 7 \cdot 7$

## 3-6a

## Problem-Solving Strategy <br> A Preview of Lesson 3-6

Look for a Pattern

## What You'll LEARN

Solve problems using the look for a pattern strategy.

We've already saved \$155 in four months. If we keep saving our money at the same rate, how long do you think it will take to save enough money to buy a DVD player that costs $\$ 330$ ?

I found the table where we listed our savings each month. Let's look for a pattern to figure it out.

We began with $\$ 50$ and added more money to our savings every month. We need to find the number of months when we will have $\$ 330$ to buy the DVD player.
Let's look for a pattern or rule that increases the balance each month. Then use the rule to extend the pattern and find the solution.

After the initial $\$ 50$, we saved $\$ 35$ per month.
Solve

Examine
Explore

Plan

## Analyze the Strategy

 To extend the pattern, add $\$ 35$ to each monthly balance until the balance equals $\$ 330$. We will have enough money saved after 9 months.We saved about $2 \times \$ 155$, or $\$ 310$ in 8 months. So, 9 months is a reasonable answer.

1. Explain when you would use the look for a pattern strategy to solve a problem.
2. Describe how to solve a problem using the look for a pattern method as a problem-solving strategy.
3. Write a problem that could be solved by looking for a pattern. Explain your answer.

| Month | Balance |
| :---: | :---: |
| 1 | $\$ 50$ |
| 2 | $\$ 85$ |
| 3 | $\$ 120$ |
| 4 | $\$ 155$ |


| Month | Balance |
| :---: | :---: |
| 1 | $\$ 50$ |
| 2 | $\$ 85$ |
| 3 | $\$ 120$ |
| 4 | $\$ 155$ |
| 5 | $\$ 190$ |
| 6 | $\$ 225$ |
| 7 | $\$ 260$ |
| 8 | $\$ 295$ |
| 9 | $\$ 330$ |

## Apply the Strategy

## Solve. Use the look for a pattern strategy.

4. LIFE SCIENCE The table shows about how many times a firefly flashes at different temperatures.
Estimate how many times a firefly will flash when the temperature is $36^{\circ} \mathrm{C}$.

| outside <br> Temperature $\left({ }^{\circ} \mathrm{C}\right)$ | Flashes per <br> Minute |
| :---: | :---: |
| 16 | $\mathbf{8}$ |
| 20 | 9 |
| 24 | 11 |
| 28 | 14 |

5. CEREAL A display of cereal boxes is stacked in the shape of a pyramid. There are 4 boxes in the top row, 6 boxes in the second row, 8 boxes in the next row, and so on. The display contains 7 rows of boxes. How many boxes are in the display?

## Mixed Problem Solving

## Solve. Use any strategy.

LIFE SCIENCE For Exercises 6-8, use the information and the graph.
6. What does 0 on this graph Summer Rainfall represent?
7. Write an integer to represent the rainfall for each (compared to normal)
 month.
8. Write a sentence that summarizes the message this graph conveys about this summer's rainfall.
9. EARTH SCIENCE Hydrothermal vents are similar to geysers, but are found on the ocean floor. A hydrothermal vent chimney can grow at an average rate of 9 meters in 18 months. What is the average rate of growth per month?
10. MULTI STEP Francisco is on vacation and is planning to send postcards and letters to his friends. He has $\$ 3.04$ to spend on postage. A stamp for a letter costs $37 \phi$, and a stamp for a postcard costs $23 \not x$. If he is going to spend the entire $\$ 3.04$ on postage, how many postcards and letters can he send?
11. BASKETBALL Laura makes 3 free throws out of every 5 she attempts. Find the number of free throws she will make after 15,20 , and 30 attempts.
12. COINS Olivia has seven coins that total $\$ 1.32$. What are the coins?
13. FOOD The school cafeteria added a breakfast special to their menu. The table shows the foods that are part of the special and the number of Calories. Estimate how many Calories there are in the special.

| Food | Calories |
| :--- | ---: |
| whole-wheat bagel | 156 |
| skim milk | 90 |
| nonfat strawberry yogurt | 183 |
| fresh fruit salad | 68 |

## 14. STANDARDIZED

 TEST PRACTICEThe total land area of Illinois is about 55,593 square miles. According to the 2000 U.S. Census Bureau, about 223.4 persons per square mile were living in Illinois. What was the approximate population of Illinois in 2000?
(A) 124,000
(B) $1,240,000$
(C) $12,400,000$
(D) $124,000,000$

## 3-6 Multiplying Integers

## H2NOSON

Mini Lab

## Materials

## What You'll LEARN

Multiply integers.

## Work with a partner.

Counters can be used to multiply positive and negative integers.


1. Write a multiplication sentence that describes the model above.

Find each product using counters.
2. $3(-2)$
3. $4(-3)$
4. $1(-7)$
5. $5(-2)$
6. Write a rule for finding the sign of the product of a positive and negative integer.

Remember that multiplication is the same as repeated addition. The multiplication expression $4(-2)$ in the Mini Lab means that -2 is used as an addend four times.
$\begin{aligned} 4(-2) & =(-2)+(-2)+(-2)+(-2) \\ & =-8\end{aligned}$


By the Commutative Property of Multiplication, $4(-2)=-2(4)$. When two integers have different signs, the following rule applies.

## N(0) tailissil Key Concept: Multiply Integers with Different Signs

Words The product of two integers with different signs is negative.
Examples 6(-4) $=-24 \quad-5(7)=-35$

## EXAMPLES Multiply Integers with Different Signs

Multiply.
(1) $3(-5)$
$3(-5)=-15$ The integers have different signs. The product is negative.
(2) $-6(8)$
$-6(8)=-48$ The integers have different signs. The product is negative.

Zero The
Multiplicative Property of Zero states that when any number is multiplied by zero, the product is zero.

Look Back You can review exponents in Lesson 1-2.

The product of two positive integers is positive. You can use a pattern to find the sign of the product of two negative integers.


When two integers have the same sign, the following rule applies.

## Not tailifesil Key Concept: Multiply Integers with Same Sign <br> Words The product of two integers with the same sign is positive. <br> Examples 2(6) $=12 \quad-10(-6)=60$

## EXAMPLES Multiply Integers with the Same Sign

Multiply.
(3) $-11(-9)$
$-11(-9)=99 \quad$ The integers have the same sign. The product is positive.
(1) $(-4)^{2}$
$(-4)^{2}=(-4)(-4) \quad$ There are two factors of -4. $=16 \quad$ The product is positive.

## Your Turn Multiply.

a. $-12(-4)$
b. $(-5)^{2}$
c. $(-2)^{3}$

## EXAMPLES Simplify and Evaluate Expressions

(5) ALGEBRA Simplify - $2(3 x)$.
$-2(3 x)=(-2 \cdot 3) x \quad$ Associative Property of Multiplication
$=-6 x \quad$ Simplify.
ALGEBRA Evaluate $p q r$ if $p=-3, q=4$, and $r=-1$.
$p q r=(-3)(4)(-1) \quad$ Replace $p$ with $-3, q$ with 4 , and $r$ with -1 .
$=(-12)(-1) \quad$ Multiply -3 and 4 .
$=12 \quad$ Multiply -12 and -1.

## © Your Turn

d. Simplify $-5(2 y)$.
e. Evaluate $x y z$ if $x=-7, y=-4$, and $z=2$.

## Skill and Concept Check

1. Model the product of 2 and -3 using counters. Then write the multiplication sentence.
2. OPEN ENDED Name two integers whose product is negative.
3. NUMBER SENSE What is the sign of the product of three negative integers? Give an example.

## CUIDRO PRACTICE

## Multiply.

4. $6(-10)$
5. $-15(-3)$
6. $(-2)^{2}$

ALGEBRA Simplify each expression.
7. $-5(2 a)$
8. $3(-6 b)$
9. $-5(-9 c)$

ALGEBRA Evaluate each expression if $f=-1, g=7$, and $h=-10$.
10. $5 f$
11. fgh
12. $-h^{2}$
13. SUBMARINES A submarine is diving from the surface of the water at a rate of 125 feet per minute. What is the depth of the submarine after 7 minutes?

## Practice and Applications

Multiply.
14. $8(-13)$
15. $-16(-5)$
16. $(-9)^{2}$
18. $-7(16)$
19. $(-6)^{2}$
20. $-20(-8)$
22. $-31(-5)$
23. $11(-20)$
24. $-\left(7^{2}\right)$
17. $-10(-17)$
21. $-15(30)$
25. $(-4)^{3}$

| For Exercises | See Examples |
| :---: | :---: |
| $14-27,44$ | $1-4$ |
| $28-35$ | 5 |
| $36-43$ | 6 |
| Extra Practice |  |
| See pages 571,598 |  |

26. Find the product of -13 and 13 .
27. Find -7 squared.

ALGEBRA Simplify each expression.
28. $-3(6 c)$
29. $-7(10 d)$
30. $5(-4 e)$
31. $9(-8 f)$
32. $-2(-3 g)$
33. $-6(-4 h)$
34. $(2 x)(-3 y)$
35. $(-5 r)(2 s)$

ALGEBRA Evaluate each expression if $w=7, x=-8, y=5$, and $z=10$.
36. $-4 w$
37. $x y$
38. $-2 x z$
39. $x y z$
40. $-7 w y$
41. $-3 z^{2}$
42. $12 x^{2}$
43. $-w z^{2}$
44. VOLUNTEERING The Volunteer Club raked leaves at several senior citizens' homes in the neighborhood. If each group of three students could remove 8 cubic meters of leaves in one hour, find an integer to represent the number of cubic meters of leaves 12 students could remove in five hours.
45. PATTERNS Find the next two numbers in the pattern $1,-2,4,-8,16, \ldots$. Then describe the pattern.

GEOMETRY For Exercises 46-48, use the graph at the right.
46. Name the ordered pairs for $P, Q$, and $R$. Multiply each $x$ - and $y$-coordinate by -1 to get three new ordered pairs.
47. Graph the ordered pairs and connect them to form a new triangle. Describe its position with respect to the original triangle.
48. In which quadrant does a triangle lie if only the $y$-coordinates of
 the original triangle are multiplied by -2 ?
49. CRITICAL THINKING For what values of $n$ is $(-2)^{n}$ positive?

## SDiFal Review with Standardized Test Practice

50. MULTIPLE CHOICE An oil rig is drilling into the ground at a rate of 7 feet per minute. What integer represents the position of the oil rig after 42 minutes?
(A) -294 ft (B) -35 ft (C) 35 ft (D) 294 ft
51. MULTIPLE CHOICE Monifa has 100 shares of stock each worth $\$ 15$. If the price drops to $\$ 8$, what integer represents the change in Monifa's current investment?
(F) $-\$ 700$
(G) $-\$ 7$
(H) $\$ 7$
(1) $\$ 700$
52. GRID IN Evaluate $-6[-2(3)+0(-5)]+(-4)^{2}$.
53. Find $-25-(-33)$. (Lesson 3-5)

ALGEBRA Evaluate each expression if $x=-4, y=6$, and $z=1$. (Lesson 3-4)
54. $x+(-2)$
55. $-1+z$
56. $-15+y$
57. $x+y$
58. EARTH SCIENCE The low temperatures in degrees Fahrenheit for ten cities on January 23 were -3, 27, $13,-6,-14,36,47,52,-2$, and 0 . Order these temperatures from greatest to least. (Lesson 3-2)

MILITARY For Exercises 59 and 60, use the double-bar graph at the right. (Lesson 2-7)
59. In which age groups are there more members in the Navy than members in the Air Force?
60. About how many 36-40 year olds are in the Air Force?

## GETTING READY FOR THE NEXT LESSON

BASIC SKILL Divide.
61. $72 \div 9$
62. $120 \div 6$
63. $84 \div 21$
64. $215 \div 43$

## 3-7 Dividing Integers

## H2NOSON

Mini Lab

## What You'll LEARN

Divide integers.

## Work with a partner.

You can use counters to model division of integers. Follow these steps to find $-10 \div 5$.


## Materials

- counters
- integer mat

There are 2 negative counters in each group. So, $-10 \div 5=-2$.
Find each quotient using counters.

1. $-6 \div 2$
2. $-12 \div 3$

Division of integers is related to multiplication. When finding the quotient of two integers, you can use a related multiplication sentence.


Since multiplication and division sentences are related, you can use them to find the quotient of integers with different signs.


These related sentences lead to the following rule.

## Notetili) Ksith Key Concept: Divide Integers with Different Signs

Words The quotient of two integers with different signs is negative.
Examples $33 \div(-11)=-3 \quad-64 \div 8=-8$

## READING Math

## Division Sentence

In the division sentence $80 \div(-10)=-8$, 80 is called the dividend, -10 is called the divisor, and -8 is called the quotient.

## REAL-LIFE MATH

ASTRONOMY In January 2004, Mars Exploration Rover successfully sent signals to Earth after landing on Mars.


## EXAMPLES Dividing Integers with Different Signs

Divide.
(1) $80 \div(-10) \quad$ The integers have different signs.
$80 \div(-10)=-8$ The quotient is negative.
(2) $\frac{-55}{11}$ The integers have different signs.
$\frac{-55}{11}=-5 \quad$ The quotient is negative.

## - Your Turn Divide.

a. $20 \div(-4)$
b. $\frac{-81}{9}$
c. $-45 \div 9$

The quotient of two integers with the same sign is positive.

## Noterifilesil Key Concept: Divide Integers with the Same Sign

Words The quotient of two integers with the same sign is positive.
Examples $15 \div 5=3 \quad-21 \div(-3)=7$

## EXAMPLE Dividing Integers with the Same Sign

Divide.
(J) $\mathbf{- 1 4} \div(-7) \quad$ The integers have the same sign.
$-14 \div(-7)=2$ The quotient is positive.

## - Your Turn Divide.

d. $-24 \div(-4)$
e. $-9 \div(-3)$
f. $\frac{-28}{-7}$

## EXAMPLE Evaluate an Expression

ASTRONOMY The average surface temperature on Mars is $-81^{\circ} \mathrm{F}$.
Use the expression $\frac{5(F-32)}{9}$, where $F$ represents the number of degrees Fahrenheit, to find the temperature on Mars in degrees Celsius.

$$
\begin{aligned}
\frac{5(F-32)}{9} & =\frac{5(-81-32)}{9} & & \text { Replace } F \text { with }-81 . \\
& =\frac{5(-113)}{9} & & \text { Subtract } 32 \text { from }-81 . \\
& =\frac{-565}{9} & & \text { Multiply } 5 \text { and }-113 . \\
& \approx-62.8 & & \text { Divide. }
\end{aligned}
$$

The average temperature on the surface of Mars is about $-63^{\circ} \mathrm{C}$.

## Skill and Concept Check

1. Write two division sentences related to the multiplication sentence $-6 \cdot 7=-42$.
2. OPEN ENDED Write a division sentence. Then draw a model to show how the quotient can be found.
3. Which One Doesn't Belong? Identify the division expression whose quotient does not have the same sign as the other three. Explain your reasoning.

| $-24 \div 6$ | $-18 \div(-9)$ | $28 \div(-7)$ | $-22 \div 11$ |
| :--- | :--- | :--- | :--- |

## CUIDED PRACTICE

Divide.
4. $32 \div(-8)$
5. $-16 \div 2$
6. $-60 \div(-5)$
7. $\frac{-6}{6}$

ALGEBRA Evaluate each expression if $d=-9, e=36$, and $f=-6$.
8. $-108 \div f$
9. $e \div d$
10. $\frac{e-f}{f}$

## Practice and Applications

## HOMEWORK HELP

## Divide.

11. $-18 \div 9$
12. $50 \div(-5)$
13. $-15 \div(-3)$
14. $\frac{21}{-7}$
15. $56 \div(-8)$
16. $\frac{0}{-5}$
17. $-52 \div(-13)$
18. $-34 \div 2$
19. $\frac{90}{6}$
20. $-300 \div 25$
21. $99 \div(-99)$
22. $-184 \div(-23)$

| For Exercises | See Examples |
| :---: | :---: |
| $11-24$ | $1-3$ |
| $25-37$ | 4 |
| Extra Practice |  |
| See pages 571,598 |  |

23. Find the quotient of -65 and 13 .
24. Divide 200 by -100 .

ALGEBRA Evaluate each expression if $r=12, s=-4$, and $t=-6$.
25. $-12 \div r$
26. $72 \div t$
27. $r \div s$
28. $r s \div 16$
29. $\frac{-r}{t}$
30. $\frac{16-(-r)}{-s}$
31. $t^{2} \div r$
32. $\frac{r^{2}}{s^{2}}$
33. FOOTBALL During the fourth quarter, the Colts were penalized 3 times for the same amount for a total of 45 yards. Write a division sentence to represent this situation. Then find the number of yards for each penalty.
34. EARTH SCIENCE Use the expression $\frac{5(F-32)}{9}$, where $F$ represents the number of degrees Fahrenheit, to convert $5^{\circ} \mathrm{F}$ to degrees Celsius.
35. PATTERNS Find the next two numbers in the pattern $729,-243,81$, $-27,9, \ldots$. Explain your reasoning.
36. SALES The graph shows five magazines that had losses in a recent year. The numbers represent the profit the magazines made in 2006 compared to 2005. What is the mean of the losses for these five magazines?
37. MULTI STEP The sea otter population is increasing. There were 2,377 sea otters in 1995. The population rose to 2,505 in 2003. Find the average rate of change for the sea otter population between 1995 and 2003.

Magazines

38. WRITE A PROBLEM Write about a situation in your life in which you used positive and negative integers. Create a problem and solve it using any of the four operations.
39. CRITICAL THINKING List all of the numbers by which -20 is divisible.

## Spiral Review with Standardized Test Practice

40. MULTIPLE CHOICE Shenequa is driving cross-country. If she expects to drive between 350 and 450 miles per day, which number of days is reasonable for her to drive 3,800 miles?
(A) fewer than 6 days
(B) between 6 and 8 days
(C) between 9 and 11 days
(D) more than 11 days
41. MULTIPLE CHOICE The width of a beach at 8:00 p.m. is 107 feet. At 3:00 A.M., the width of the beach narrows to 23 feet due to the high tide. What is the average rate of change per hour of the beach width?
(F) $-15 \mathrm{ft} / \mathrm{h}$
(G) $-13 \mathrm{ft} / \mathrm{h}$
(H) $-12 \mathrm{ft} / \mathrm{h}$
(I) $-10 \mathrm{ft} / \mathrm{h}$
42. SHORT RESPONSE Find the mean of the following low temperatures for a 5 -day period: $-2^{\circ} \mathrm{F},-3^{\circ} \mathrm{F}, 1^{\circ} \mathrm{F}, 4^{\circ} \mathrm{F}$, and $-5^{\circ} \mathrm{F}$.

Multiply. (Lesson 3-6)
43. $14(-2)$
44. $(-9)^{2}$
45. $-20(-3)$
46. $-5(7)$
47. Find $6-(-12)$. (Lesson 3-5)
48. DIVING Mark jumped into 12 feet of water and touched the bottom of the pool before he surfaced. Which integer describes where Mark was in relation to the surface of the water? (Lesson 3-1)
49. PHYSICAL SCIENCE A chemistry experiment requires 3 milligrams of potassium chloride. How many grams of potassium chloride are needed? (Lesson 1-8)

## Study Guide and Review

## Vocabulary and Concept Check

absolute value (p. 107)
additive inverse (p. 121)
coordinate grid (p. 112) coordinate plane (p. 112) graph (p. 106) integer (p. 106)
negative integer (p. 106)
opposite (p. 121)
ordered pair (p. 112)
origin (p. 112)
positive integer (p. 106)
quadrant (p. 113)
$x$-axis (p. 112)
$x$-coordinate (p. 112)
$y$-axis (p. 112)
$y$-coordinate (p. 112)

## Choose the correct term or number to complete each sentence.

1. Integers less than zero are (positive, negative) integers.
2. Two numbers represented by points that are the same distance from 0 are (opposites, integers).
3. The absolute value of 7 is $(7,-7)$.
4. The opposite of $(-12,12)$ is -12 .
5. The (coordinate plane, origin) is the point where the horizontal and vertical number lines intersect.
6. The $x$-axis and the $y$-axis separate the plane into four (quadrants, coordinates).
7. The first number in an ordered pair is the ( $x$-coordinate, $y$-coordinate).
8. The sum of two (positive, negative) integers is negative.
9. The product of a positive and a negative integer is (positive, negative).
10. The quotient of a negative integer and a (positive, negative) integer is negative.

## Lesson-by-Lesson Exercises and Examples

3-1 Integers and Absolute Value (pp. 106-108)
Write an integer for each situation.
11. a loss of $\$ 150$
12. 350 feet above sea level
13. a gain of 8 yards
14. $12^{\circ} \mathrm{F}$ below 0

Evaluate each expression.
15. $|-11|$
16. $|100|$
17. $|5|$
18. $|-32|$
19. $|-16|+|9|$

## Example 1 Write an integer for 8 feet

 below sea level.Since this situation represents an elevation below sea level, -8 represents the situation.

Example 2 Evaluate $|-10|$.
On the number line, the graph of -10 is 10 units from 0 .


So, $|-10|=10$.

## 3-2 Comparing and Ordering Integers (pp. 109-111)

Replace each with $<$ or $>$ to make a true sentence.
20. $-3--9$
21. $8-12$
22. -3 - 3
23. $|-10| \cdot|-13|$
24. $25 \bigcirc|8|$
25. $0 \bullet|-4|$

Order each set of integers from least to greatest.
26. $-3,8,-10,0,5,-12,9$
27. $-21,19,-23,14,-32,25$
28. $|-17|,-18,18,|15|,-16,|16|$
29. EARTH SCIENCE The predicted low temperatures for Monday through Friday are $3^{\circ},-1^{\circ},-2^{\circ}, 0^{\circ}$, and $1^{\circ}$. Order the temperatures from greatest to least.

Example 3 Replace with $<$ or $>$ to make $-4-7$ a true sentence.
Graph each integer on a number line.


Since -4 is to the right of $-7,-4>-7$.
Example 4 Order the integers $-4,-3$, $5,3,0,-2$ from least to greatest.
Graph the integers on a number line.


Order the integers by reading from left to right: $-4,-3,-2,0,3,5$.

## 3-3 Geometry: The Coordinate Plane (pp. 112-115)

Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.
30. $A$
32. $C$

On graph paper, draw a coordinate plane. Then graph and label each point.
34. $E(1,-4)$
35. $F(-5,2)$
37. $H(4,0)$
31. $B$
33. $D$

36. $G(-2,-3)$ $\qquad$

Example 5 Name the ordered pair for point $W$ graphed at the right. Then identify the quadrant in which point $W$ lies.


The ordered pair is $(-4,-5)$. Point $W$ is in quadrant III.

Example 6 Graph and label the point $S(3,-1)$.

Draw a coordinate plane. Move 3 units to the right. Then move 1 unit down. Draw a dot and label it $S(3,-1)$.


## 3-4. Adding Integers (pp. 120-124)

## Add.

38. $-6+8$
39. $-4+(-9)$
40. $7+(-12)$
41. $-18+18$
42. FOOTBALL On the first play of the game, the Bulldogs lost 8 yards. On the second and third plays, they gained 5 yards and then lost 2 yards. Find the result of the first three plays.

## 3-5 Subtracting Integers (pp. 128-131)

## Subtract.

43. $-5-8$
44. $3-6$
45. $5-(-2)$
46. $-4-(-8)$

## 3-6 Multiplying Integers (pp. 134-137)

## Multiply.

47. $-4(3)$
48. $8(-6)$
49. $-5(-7)$
50. $-2(40)$

ALGEBRA Evaluate each expression if $a=-4, b=-7$, and $c=5$.
51. $a b$
52. $-3 c$
53. $b c$
54. $a b c$

## 3-7 Dividing Integers (pp. 138-141)

## Divide.

55. $-45 \div(-9)$
56. $36 \div(-12)$
57. $-12 \div 6$
58. $-81 \div(-9)$
59. HIKING Marta started a hike at sea level and ended the hike 6 hours later at 300 feet below sea level. If Marta hiked at the same pace during the trip, how far did she travel each hour?

## Example 7 Find -4 +3 .



So, $-4+3=-1$.

Example 8 Find - $\mathbf{3 - 9}$.

$$
\begin{aligned}
-3-9 & =-3+(-9) & & \text { To subtract 9, add }-9 . \\
& =-12 & & \text { Simplify. }
\end{aligned}
$$

Example 9 Find -4(3).
$-4(3)=-12$ The integers have different signs. The product is negative.

Example 10 Evaluate $x y z$ if $x=-6$, $y=11$, and $z=-10$.

$$
\begin{aligned}
x y z & =(-6)(11)(-10) & & x=-6, y=11, z=-10 \\
& =(-66)(-10) & & \text { Multiply }-6 \text { and } 11 . \\
& =660 & & \text { Multiply }-66 \text { and }-10 .
\end{aligned}
$$

Example 11 Find -72 $\div(-9)$.
$-72 \div(-9)=8$ The integers have the same sign. The quotient is positive.

## Practice Test

## Vocabulary and Concepts

1. Explain what it means for two numbers to be opposites.
2. Name the rule for dividing integers with different signs.

## Skills and Applications

Write an integer for each situation.
3. a stock increased by $\$ 5$
4. 1000 в.c.
5. an elevator goes down
11 floors

Replace each with $<$ or $>$ to make a true sentence.
6. $-3-9$
7. $|9| \ominus|-12|$
8. $|-7| \bigcirc 9$
9. WEATHER The local weather service records the following changes in temperature during the last week: $4,-7,-3,2,9,-8,1$. Order these temperature changes from greatest to least.

Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.
10. $P$
11. $Q$
12. $R$


Add, subtract, multiply, or divide.
13. $-3+8$
14. $12+(-19)$
15. $-3-8$
16. $-7-(-20)$
17. $-7(-3)$
18. $5(-11)$
19. $-24 \div 8$
20. $-36 \div(-9)$

ALGEBRA Evaluate each expression if $a=-5, b=4$, and $c=-12$.
21. $c-a$
22. $a b$
23. $a c \div b$
24. STOCK MARKET The value of a stock went down $\$ 3$ each week for a period of seven weeks. Describe the change in the value of the stock at the end of the seven week period.

## Standardized Test Practice

25. MULTIPLE CHOICE Choose the graph that shows the ordered pair $(2,-1)$.
(A)

(B)

(C)

(D)


## PART 1 Malitiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The table shows the five most common languages spoken in the United States other than English.
How many more people speak
Chinese than French?
(Lesson 1-1)

| Language | Number <br> of People |
| :--- | ---: |
| Spanish | $26,745,067$ |
| Chinese | $1,976,564$ |
| French | $1,914,918$ |
| German | $1,224,213$ |
| Tagalog | $1,184,939$ |

Source: U.S. Census Bureau
(A) $\mathbf{6 1 , 6 4 6}$
(B) $1,914,918$
(C) $1,976,564$
(D) $3,891,482$
2. Which is equivalent to $5^{4}$ ? (Lesson 1-2)
(F) 20
(G) 125
(H) $5 \cdot 5 \cdot 5 \cdot 5$
(1) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
3. How many millimeters are in 13 centimeters? (Lesson 1-8)
(A) 0.13
(B) 1.3
(C) 13
(D) 130
4. Salvador recorded the number of minutes it took him to drive to work each day for a week. Find the mean for the following times: 12, 23, 10, 14, and 11. (Lesson 2-4)
(F) 12 min
(G) 13 min
(H) 14 min
(I) 15 min
5. Find the interquartile range of the data in the box-and-whisker plot. (Lesson 2-6)


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6. What is the value of $|-2|$ ? (Lesson 3-1)
(F) -2
(G) -1
(H) 0
(I) 2
7. Which of these is the correct order of the integers from greatest to least? (Lesson 3-2)

$$
\begin{array}{ll}
\text { (A) } 0,1,-2,-5 & \text { (B) } 1,0,-2,-5 \\
\text { (C) }-5,-2,0,1 & \text { (D) } 1,-2,-5,0
\end{array}
$$

8. Which of these are the coordinates of point Z? (Lesson 3-3)
(F) $(2,2)$
(G) $(3,3)$
(H) $(-2,-2)$
(I) $(-3,-3)$

9. If you graph and connect the following points on a coordinate plane, what shape would you make? (Lesson 3-3)

$$
(0,-3),(0,4),(4,4),(4,-3)
$$

(A) rectangle
(B) triangle
(C) circle
(D) parallelogram
$\because$..10. Which of the following points on the number line represents opposites?
(Lesson 3-4)

(F) $C$ and $D$
(H) $A$ and $D$
(G) $B$ and $D$
(I) A and C

## Fit: TAMinc IIP

Question 10 Always be sure to check every answer choice of a multiple-choice question. Start with answer choice $F$. Each time you find an incorrect answer choice, cross it off so you remember that you've eliminated it.

## PART 2 shons Response/Grad in

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.
11. Wallpaper costs $\$ 16$ per roll, and border costs $\$ 9$ per roll. If 12 rolls of wallpaper and 6 rolls of border are needed for one room, find the total cost of the wallpaper and border. (Lesson 1-3)
12. Each triangle in the figure below is made from three toothpicks. Extend the pattern. Find the number of toothpicks in the fifth figure. (Lesson 1-7)

13. Write 63.5 million in scientific notation.
(Lesson 1-9)
For Questions 14 and 15, use the information below.
The graph shows the number of orders taken each hour one day at a fast food restaurant. (Lesson 2-2)

14. About how many orders were taken at 11:00 A.м.?
15. What time of the day appears to be the least busy at this restaurant?

For Questions 16 and 17, use the information below.
The temperature of the liquid in Connor's beaker changed drastically. The temperatures he recorded were $-28^{\circ} \mathrm{F}, 59^{\circ} \mathrm{F}, 1^{\circ} \mathrm{F},-16^{\circ} \mathrm{F}$, $24^{\circ} \mathrm{F}$, and $8^{\circ} \mathrm{F}$. (Lesson 3-2)
16. Order the temperatures from least to greatest.
17. Find the mean temperature.
18. When Elena went hiking at 8 A.M., she started at an elevation of 16 meters below sea level. But at 2 P.m., she was 12 meters higher than when she started. What was Elena's elevation at 2 P.m.? (Lesson 3-5)
19. Find $\frac{-12+8(-3)}{-2-7}$. (Lesson 3-7)

## PART 3 Extmaded Response

Record your answers on a sheet of paper. Show your work.
20. Use the table of ordered pairs at the right to answer the questions below. (Lesson 3-3)
a. Plot the ordered pairs in

| $x$ | $y$ |
| ---: | ---: |
| -2 | 1 |
| 3 | 2 |
| -1 | -4 | the table on a coordinate plane.

b. In which quadrant is a point on the graph not represented?
c. Connect the points on the graph. What shape do they form?
d. How would you double the size of the figure you drew above?
e. Add four more columns to the table. Label the columns $x+y, x-y, x y$, and $x \div y$. Complete the table.

